International Journal of Scientific Advances
ISSN: 2708-7972
Volume: 2 | Issue: 1 | Jan - Feb 2021 Available Online: www.ijscia.com
DOI: 10.51542/ijscia.v2i1.5

Proposed Model for Optimal Inventory Policy

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ABSTRACT

The topic of management of stock is an energetic importance to the success of any organization and is one of the serious determinants of the continuity and efficient productivity of any organization. The study considers significant because it is hoped that on the completion, the study will provide further insights into the understanding of stock control measures. Through using vehicle service center as a reference point, the study will make an interesting contribution to the understanding of the general and specific effects of stores control in other private and public utilities. In addition, the study will further justify the need to strengthen management and control of stock with anticipated benefit in view. This paper will be proposed a mathematical model for optimal inventory policy to minimize the total inventory cost by using the assignment technique and will be applied it on a case study.

Keywords: Inventory; Supply chain; Stock control; Assignment Technique

INTRODUCTION

Inventory systems have received extensive attention since the first half of the twentieth century. Effective management of inventory using Operations Research tools has been a major concern in both the literature and the industry. Basically, yet crucial questions such as when to replenish and how much to replenish have been the focus of inventory management. There are several reasons for maintaining inventory, the most important of which is to reduce the risk of change in the rate of demand and supply. Therefore, inventory provides the buffer between supply and demand. As it enables to provide a high level of service, which is very important for the company to stay in the market, where high inventory levels ensure greater customer satisfaction.

Inventory management is not limited to retail stores. In fact, stocks are pervasive in the business world. Inventory holding is essential for any company that deals with physical products, especially in-service field. For example, manufacturers need inventories of the materials required to make their products. They also need inventories of finished products awaiting shipment. Likewise, any company that has a warehouse need to hold inventories of goods to be available for purchase by customers or present a service.

How do companies use operations research to improve their inventory policy regarding when and how much to replenish their stock? They manage inventory through the following steps:

1. Formulate a mathematical model that describes the attitude of the inventory system.
2. determine an optimal inventory policy with respect to this model.
3. Use IT system to maintain a record of the current stock levels.
4. Using this record of current stock levels to apply the optimal inventory policy for signal when and how much to replenish inventory.

In this paper, we present a literary review about the research related to the inventory system and the methods used to solve this problem. In addition, the methodological methods for solving such a problem with mentioning the different types that affect the cost of inventory. After that, the proposed mathematical model is presented based on the assignment technique to determining the size of the supplied quantity and determining the optimal time for it, then the proposed model will be applied to one of the car service centers for a specific model of cars that can be generalized later

LITERATURE REVIEW

Libero Poulos [1] developed some analytical results on the tradeoff between FG inventory and advance demand information ADI for a model of a single-stage, make-to-stock supplier who uses an order base stock replenishment policy to meet customer orders that arrive a fixed time in advance of their due dates. **Zhou et al [2]** studied a single-product periodic-review inventory system with multiple types of returns. The serviceable products used to fulfill stochastic customer demand can be either manufactured/ordered, or remanufactured from the returned products, and the objective was to minimize the expected total discounted cost over a finite planning horizon. It showed that, under some circumstances but not all, the optimal policy had a simple form and can be completely characterized by a sequence of constant control parameters. However, in some other scenarios, the optimal policy can be quite complicated and control parameters are state-dependent. It presented a partial characterization on the optimal control policy for the general case when there are only two types of returns. **Tripathy and Pattnaik [3]** investigated an instantaneous production plan to obtain an optimal ordering policy wherein demand exceeds supply, all items were subjected to inspection and defective items were discarded. The unit cost of production was inversely related to both process reliability and demand rate. The market was able to absorb virtually any quantity of the product rolled out from the production line. This situation was typical of a technologically advanced product entering the growth phase of its life cycle. Since the demand for the product is high, the manufacturer will increase production to meet it, which will result in lower unit cost of production because production overheads are spreaded over the items. **Oseph et al. [4]** considered a multi-level/multi-machine lot sizing problem with flexible production sequences, where the quantity and combination of items required to produce another item need not be unique. The problem is formulated as a mixed-integer linear program and the notion of echelon inventory is used to construct a new class of valid inequalities, which are called echelon cuts. Numerical results show the computational power of the echelon cuts in a branch-and-cut algorithm. **Jong et al. [5]** allowed the backorder rate as a control variable to widen applications of a continuous review inventory model. Moreover, it presented a new form for backorder rate that was dependent on the amount of shortages and backorder price discounts. Besides, it also treated the ordering cost as a decision variable. Hence, it developed an algorithmic procedure to find the optimal inventory policy by mini-max criterion. Finally, a numerical example is also given to illustrate the results. **Huaming et al. [6]** assumed that the replenishment lead time is dependent on both lot size of the buyer and production rate of the vendor, an integrated production inventory model is presented. The decision-making interaction of lead-time between a buyer and a vendor in the integrated inventory model is analyzed. In terms of the model, a solution procedure has been developed to obtain the efficient ordering strategy for a manufacturing company. The numerical examples are employed to validate the solution procedure. **Wei et al. [7]** studied a single-period inventory problem with discrete stochastic demand. Most of their works were based on the expected profit/cost criterion or expected utility criterion. It considered the effect of irrational factor under uncertainty and therefore in- corporate prospect theory into inventory model. Their objective is to maximize the overall value of the prospect, which can be calculated by using the value function and the weighting function. **Roushdy [7]** developed a mixed integer-programming model in order to the production and inventory decisions through the supply chain of various entities. The main objective of this paper was determining the warehouse allocation in the supply chain to get the minimum inventory cost for the system.

The above review has shown that almost the efforts which have been done is directed towards the studying of inventory control with the assumption data with some applications, whereas real date has given little considerations. Various authors have exposed their work for the system modeling with some applications; their contributions were limited. However, the review has established that, there is a need for more investigation on the inventory control with the real data. Consequently, there is a general attention to establish suitable concept by which the inventory control with respect of planning (scheduled) maintenance has to be established.

METHODOLOGY

In general, the inventory management search about answer for two simple questions: "when to order?" and "how much to order?" to optimize the total inventory cost. Therefore, this section will illustrate the answer about these questions by the proposed mathematical model in this paper. Before presenting the proposed model, we must know the parameters that affect the total inventory cost.

1. The inventory level

An inventory is represented in the simple diagram of Figure 1. Items flow into the system, remain for a time and then flow out. Inventories occur whenever the time an individual enters is different than when it leaves. During the intervening interval, the item is part of the inventory

FIGURE 1: A system component with inventory

The inventory level depends on the relative rates of flow in and out of the system. Define *y(t)* as the rate of input flow at time t and *Y(t)* the cumulative flow into the system. Define *z(t)* as the rate of output flow at time t and *Z(t)* as the cumulative flow out of the system. The inventory level, *I(t)* is the cumulative input less the cumulative output.

$$I(t) = Y(t) - Z(t) = \int_0^t y(x)dx - \int_0^t z(x)dx \qquad (1)$$

Figure 2 represents the inventory system when the rates vary with time. The figure might represent a raw material inventory. The flow out of inventory is a relatively continuous activity where individual items are placed -into the production system for processing. To replenish the inventory, an order is placed to a supplier. After some delay time, called the *lead-time*, the raw material is delivered in a *lot* of a specified amount. At the moment of delivery, the rate of input is infinite and at other times it is zero. Whenever the instantaneous rates of input and output to a component are not the same, the inventory level changes. When the input rate is higher, inventory grows; when output rate is higher, inventory declines.

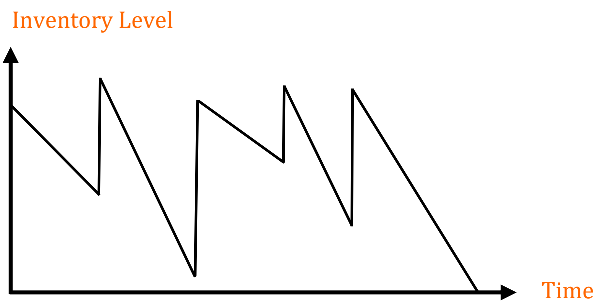

FIGURE 2: Inventory Fluctuations as a Function with time

2. Inventory Costs

Inventory costing is defined as the total cost that a company faces while holding inventory, and it is often one of the most important factors in the success of a business. Inventory cost control has many aspects, including finance, equipment, labor, preventive measures, insurance, handling, obsolescence, misappropriation losses, and the opportunity cost of choosing to deal with inventory. All of these factors combine to create the total cost of carrying inventory. Therefore, we can classify the total inventory costs to carrying cost, ordering cost, and stock-out cost

2.1 Carrying Cost

Inventory carrying cost includes warehousing and inventory management as inventory management and process involves extensive use of building, material handling equipment, IT software applications, hardware equipment along with operations and management Staff resources.

- ***Storage Cost***

Inventory storage costs usually include the cost of the building's rental, facility maintenance, and related costs. Such as, he cost of material handling equipment, IT equipment and applications, including the cost of purchase, depreciation, lease or rental as the case may be. Additional costs include operational costs, consumables, communication and utility costs, along with the cost of human resources used in operations as well as administration

- ***Capital Cost***

It includes investment costs, interest on working capital, taxes on paid inventory, insurance costs and other costs associated with legal requirements.

2.2 Ordering Cost

Ordering costs, also known as setup costs, are costs that are incurred every time you place an order from your supplier.

2.3 Stock-out Cost

These costs, also called shortage costs, occur when companies run out of inventory for any reason.

- Production disruption - When a business involves producing goods in addition to selling them, the shortfall means that the company will have to pay for things like unemployed workers and factory expenses, even when nothing is produced

- Emergency Shipments - For retailers, running out of stock can mean paying extra to get a shipment on time, or changing suppliers

- Customer Loyalty and Reputation - Aside from losing business from clients who go elsewhere to make purchases, the company is damaged by customer loyalty and reputation when their customers are unhappy.

INVENTORY MODELS

There are many models from the inventory problems but in this section, we will present a lot size model without shortages and lot size model with shortages only and after that we will present a proposed inventory model to minimize the total inventory cost using the assignment technique.

1. Lot Size Model with no Shortages

The assumptions of the model are described as Figure 3, which shows a plot of inventory level as a function of time. The inventory level ranges between 0 and the amount Q. The fact that it never goes below 0 indicates that no shortages are allowed. Periodically an order is placed for replenishment of the inventory. The order quantity is Q. The arrival of the order is assumed to occur instantaneously, causing the inventory level to shoot from 0 to the amount Q. Between orders the inventory decreases at a constant rate a. The time between orders is called the cycle time, t, and is the time required to use up the amount of the order quantity.

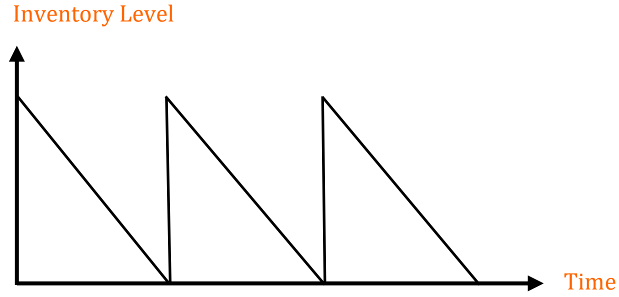

FIGURE 3: Lot Size Model with no Shortages

The total inventory cost expressed is
Total Inventory Cost = Ordering Cost + Purchase Cost + Carrying Cost

$$T = \frac{D.K}{Q} + D.P + \frac{H.Q}{2} \quad (2)$$

Where
D: Demand per year
K: Setup cost per order
H: Carrying cost per unit
P: purchase cost per unit
Q: required quantity per order

2. Lot Size Model with Shortages or Backorder

A deterministic model considered in this section allows shortages to be backordered. This situation is illustrated in Fig. 6. In this model the inventory level decreases below the 0 level. This implies that a portion of the demand is backlogged. A backorder is represented in the figure by a negative inventory level.

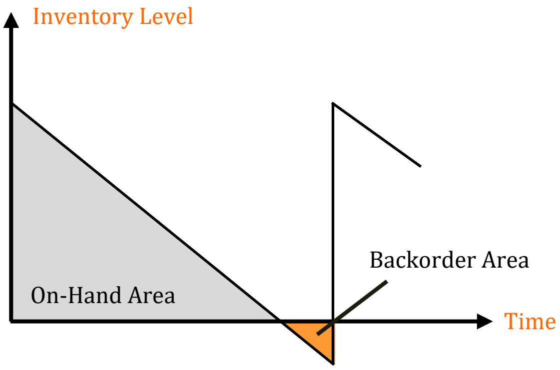

FIGURE 4: Lot Model with Shortage

The total inventory cost is
Total inventory Cost = Setup cost + Purchase cost + Holding cost + Backorder cost

In the next section, we will introduce the mathematical model for inventory system as mixed-integer programming problem and this model will allowable shortage in stock level to minimize the total inventory cost.

3. Mathematical Model of Inventory problem for multi items.

In this section, we will illustrate how the proposed model for solving the inventory problem is built as an assignment model to minimize the total inventory cost. Figure 5 illustrates that above the diagonal of the table represents the carrying or holding cost for the item, while under the diagonal of the table represents the Shortage or backorder cost.

period	1	2	.	.	M
1					
2				Holding cost	
.					
.		Backorder cost			
M					
Demand	d_1	d_2	.	.	d_M

FIGURE 5: Inventory cost problem as assignment problem model

Model assumptions

1. The demand varies with the time and known.
2. Purchase price per any unit is constant though the plan period
3. The required units to satisfy demand in a particular period can be acquired at any time including the backorders.
4. Backorder cost may vary with the time.
5. The replenishment lead-time is known with certainty so that delivery can be timed to occur accordingly.
6. The unit variable cost does not depend upon replenishment quantity i.e. no quantity discounts are permitted.
7. If the required quantity is disbursed in the same period, then the carrying (holding) and backorder (shortage) cost equal zero.

Item No. n	Period					
Period	1	2	3	.	.	m
1	0	h_{12}^{n}	h_{13}^{n}	.	.	h_{1m}^{n}
2	b_{21}^{n}	0	h_{23}^{n}	.	.	h_{2m}^{n}
						.
						.
						0

Item No.1	Period					
Period	1	2	3	.	.	m
1	0	h_{12}^{1}	h_{13}^{1}	.	.	h_{1m}^{1}
2	b_{21}^{1}	0	h_{23}^{1}	.	.	h_{2m}^{1}
						.
						.
						0

Item No. 2	Period					
Period	1	2	3	.	.	m
1	0	h_{12}^{2}	h_{13}^{2}	.	.	h_{1m}^{2}
2	b_{21}^{2}	0	h_{23}^{2}	.	.	h_{2m}^{2}
.	.	.	.	.	.	.
.	.	.	.	.	.	.
m	b_{m1}^{2}	b_{m2}^{2}	b_{m3}^{2}	.	.	0

FIGURE 6: The inventory Cost matrix for all items

Figure. 6 represents the sequence of the inventory cost problem that means it can makes the order at period i this quantity may be put in the store for multi-periods. In this case the cost will take in consideration called carrying cost. On the other hand if the responsible person makes order in period i but this order is received in another planned time say in period j. In this case the cost becomes the backorder cost.

The inventory problem can be formulated as an assignment model. The inventory problem can be considered as transportation model with multi sources and multi demand. The source of the each period must be attained all required in planned horizon and the demand of each period is already known. The objective function is minimization the total inventory cost whereas the total inventory cost equals the summation of the carrying (holding) cost, backorder (shortage) cost and ordering (setup) cost.

The first constraint set is put to attain the required demand through the planned horizon. The second constraint set to be confirmed that all required demand at each period will deliver. On the other hand, the third constraint is put to confirm the order or not.

The objective function for Minimization of the total inventory cost

$$Min \sum_{i=1}^{m}\sum_{j=i+1}^{m}\sum_{k=1}^{n} h_{ijk}x_{ijk} + \sum_{i=2}^{m}\sum_{j=1}^{i-1}\sum_{k=1}^{n} b_{ijk}x_{ijk} + \sum_{i=1}^{m} S_i y_i \qquad (3)$$

Subject to

$$\sum_{j=1}^{M} x_{ijk} \leq Q_k . y_i \qquad i = 1,2, \ldots, M \ \& \ k = 1,2, \ldots, N \qquad (4)$$

$$\sum_{i=1}^{M} x_{ijk} \leq d_{jk} \qquad j = 1,2, \ldots, M \ \& \ k = 1,2, \ldots, N \qquad (5)$$

$$y_i = 0,1 \qquad i=1,2,\ldots,M \qquad (6)$$

$$x_{ijk} \geq 0 \text{, } Integer \qquad (7)$$

Where

x_{ijk} represents the number of units acquired in period i for demand in period *j* (associated with **x,** is used. the holding cost when i < j. and backordering cost when *i > j*) for item *k*

H represents the holding cost (in EGP) per unit per period.

S represents setup cost or ordering cost (in EGP)

M represents the length of planning horizon. (i.e. the number of periods in planning horizon)

N represents the number of items.

b represents the backorder cost (in EGP) per unit per period

d represents the number of the required units,

y_i = 0 if x_{ijk} = 0. (i.e., if no replenishment is made in period i. for all j & all item *k*)
=1 if x_{ijk} > 0. (i.e., if replenishment is made in period i. for all j & all item *k*)

Q = a large number ≥ $d_1 + d_2 + \ldots + d_M$

APPLICATION

A case study was conducted to apply the inventory model for a service center for a well-known brand of vehicles in Egypt. This service center performs regular maintenance for specific brand car model. This application displays the inventory policy to provide the maintenance department in the service center with the spare parts required for the periodic maintenance schedule for the brand car being studied. The service center makes a purchase order by the required quantities from the spare parts every two months (i.e. six purchase orders per year).

1. Collected Data

The automotive service station is interested in determining when to receive a batch of spare parts and how many quantities from spare parts in each batch taken in consideration several costs will be presented later.

From the back history of fast-moving items for many years that require the provision of spare parts necessary to carry out regular maintenance at the service station of a car of a selected brand for a period of six periods that represent a year, we can summarize the average required quantities from different spare parts in Table 1.

TABLE 1: The required spare parts for carry out service to each item per period

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Oil Filter	145	170	175	180	130	135
Spark Plug	260	270	320	320	260	240
Air Filter	70	70	80	80	70	60
Engine belt	30	40	30	40	40	30
Fuel Filter	170	200	200	210	180	160
Front brake pads	430	510	525	550	450	400
Rear brake pads	330	400	375	420	420	300

The service station policy prohibits deliberate planning of deficiency of any of its components. However, there are occasional shortages of some spare parts, and it is estimated that every spare part that is unavailable cause extra costs an amount of money per month. This deficiency cost includes the additional cost of supplying replacement parts from the local market at an otherwise higher price, the loss of interest due to the increased cost of sales, the cost of keeping additional records, etc. On the other hands, each time a payment is requested, a setup cost is incurred. This cost includes "processing" costs, administrative costs, transportation, record keeping, etc.

Note that this cost requires ordering parts in large batches. Ordering spare parts in large batches lead-to a large inventory. The holding cost of keeping the inventory includes the cost of capital tied to inventory and because this money has been invested in inventory it cannot be used in other investment ways, this cost of capital is made up of lost return (referred to as opportunity cost). Other components of the cost of tenure include the cost of renting the storage space, the cost of insuring against loss of inventory due to fire, theft, or vandalism, taxes based on the value of the inventory, and the cost of the personnel who oversee and protect the inventory. Inventory costs are presented in the table 2

TABLE 2: Holding cost for required items through holding periods

	One period	Two periods	Three periods	Four periods	Five periods
Oil Filter	29	58	87	116	145
Spark Plug	20	40	60	80	100
Air Filter	23	46	69	92	115
Engine belt	28	56	84	112	140
Fuel Filter	13	26	39	52	65
Front brake pads	41	82	123	164	205
Rear brake pads	45	90	135	180	225

TABLE 3: Backorder Cost for required items according to backorder periods

	One period	Two periods	Three periods	Four periods	Five periods
Oil Filter	48	55	63	72	83
Spark Plug	32	37	43	49	57
Air Filter	38	43	50	57	66
Engine belt	45	52	60	68	79
Fuel Filter	22	25	29	33	38
Front brake pads	67	77	88	101	116
Rear brake pads	76	87	100	115	132

TABLE 4: Ordering Cost

Period No.	Ordering Cost (EGP)
1	64000
2	64000
3	64000
4	64000
5	64000
6	64000

2. Formulation of the Inventory Practical Problem

$$Min \sum_{i=1}^{6}\sum_{j=i+1}^{6}\sum_{k=1}^{7} h_{ijk}x_{ijk} + \sum_{i=2}^{6}\sum_{j=1}^{i-1}\sum_{k=1}^{7} b_{ijk}x_{ijk} + \sum_{i=1}^{6} S_i y_i \quad (7)$$

Subject to

$$\sum_{j=1}^{6} x_{ijk} \le Q_k . y_i \qquad i = 1,2, \ldots ,6 \;\&\; k = 1,2, \ldots ,7 \quad (8)$$

$$\sum_{i=1}^{6} x_{ijk} \le d_{jk} \qquad j = 1,2, \ldots ,6 \;\&\; k = 1,2, \ldots ,7 \quad (9)$$

$$y_i = 0,1 \qquad i=1, 2,\ldots, 6 \quad (10)$$

$$x_{ijk} \ge 0 \text{ , } Integer \quad (11)$$

3. Results & Discussions

LINGO software is used for solving the problem to find the optimal inventory policy for six periods (i.e. one year), which minimizes the inventory cost for spare parts of the regular maintenance for chosen brand car. Table 5 introduces the optimal inventory policy, where the minimum total inventory cost through six periods (i.e. a year) is 368980 EGP plus purchase cost the Items

TABLE 5: Optimal Inventory Policy by Using LINGO Software

```
Global optimal solution found.
  Objective value:                    368980.0
  Objective bound:                    368980.0
  Infeasibilities:                    0.000000
  Extended solver steps:                     0
  Total solver iterations:                 643
  Elapsed runtime seconds:                0.53

  Model Class:                            MILP

  Total variables:                         264
  Nonlinear variables:                       0
  Integer variables:                         6

  Total constraints:                        49
  Nonlinear constraints:                     0

  Total nonzeros:                          732
  Nonlinear nonzeros:                        0
```

Variable	Value	Reduced Cost
X231	240.0000	0.000000
X561	260.0000	0.000000
X232	320.0000	0.000000
X562	240.0000	0.000000
X233	180.0000	0.000000
X563	180.0000	0.000000
X234	70.00000	0.000000
X564	80.00000	0.000000
X235	240.0000	0.000000
X565	260.0000	0.000000
X236	525.0000	0.000000
X566	400.0000	0.000000
X237	375.0000	0.000000
X567	300.0000	0.000000
Y1	1.000000	64000.00
Y2	1.000000	64000.00
Y4	1.000000	64000.00
Y5	1.000000	64000.00
X111	240.0000	0.000000
X112	260.0000	0.000000
X113	140.0000	0.000000
X114	60.00000	0.000000
X115	240.0000	0.000000
X116	430.0000	0.000000
X117	330.0000	0.000000
X221	280.0000	0.000000
X222	280.0000	0.000000
X223	150.0000	0.000000
X224	80.00000	0.000000
X225	280.0000	0.000000
X226	510.0000	0.000000
X227	400.0000	0.000000
X441	240.0000	0.000000
X442	320.0000	0.000000
X443	160.0000	0.000000
X444	70.00000	0.000000
X445	240.0000	0.000000
X446	550.0000	0.000000
X447	420.0000	0.000000
X551	280.0000	0.000000
X552	260.0000	0.000000
X553	150.0000	0.000000
X554	80.00000	0.000000
X555	280.0000	0.000000
X556	450.0000	0.000000
X557	420.0000	0.000000

We can summarize the optimal solution as the following: No. of orders through a year are four orders; the first order occurs at first period to cover the required quantities for the first planned period only while the second purchase order occurs at the second period to cover the required quantities for the second and third planned period . On the other hands, the third purchase order happens at the fourth period to cover the required quantities for fourth planned period only, and the last purchase order occurs at the fifth period to cover the required quantities or fifth and sixth planned period by total inventory cost equal to 368980 EGP. When comparing with the results of the proposed model by the current strategy that costs 384000 EGP for the inventory cost (i.e. six order multiples with ordering cost to avoid the holding and backorder cost), the new proposed strategy will save about 4 % from the total inventory cost.

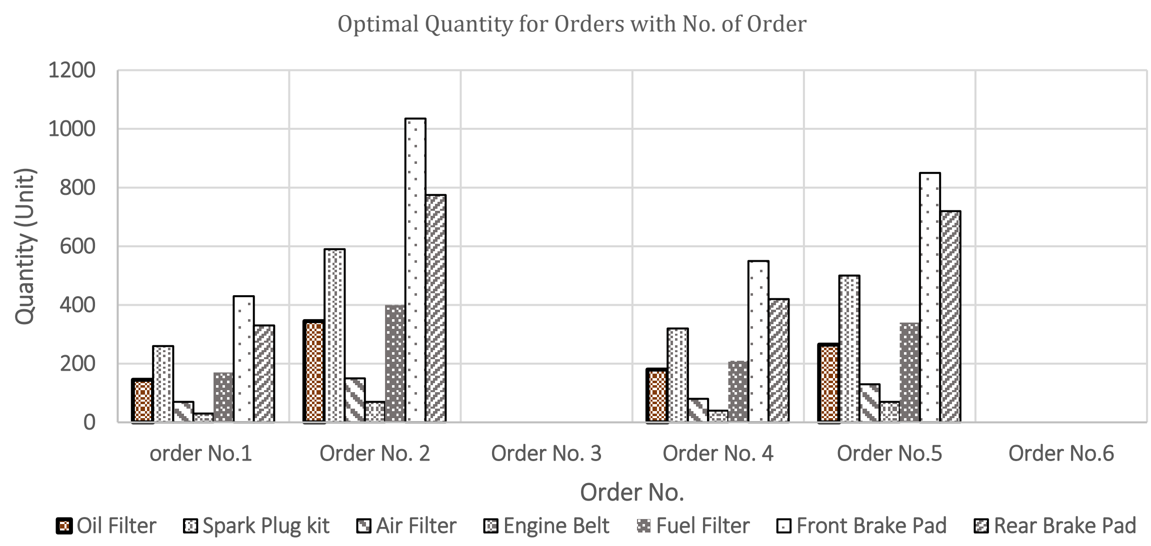

FIGURE 7: Order quantities for all Required Spare Parts According to Purchase Order No.

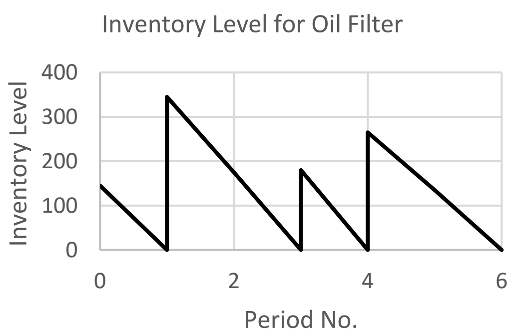

FIGURE 8: Required Quantities for Oil Filter per Purchase Order No.

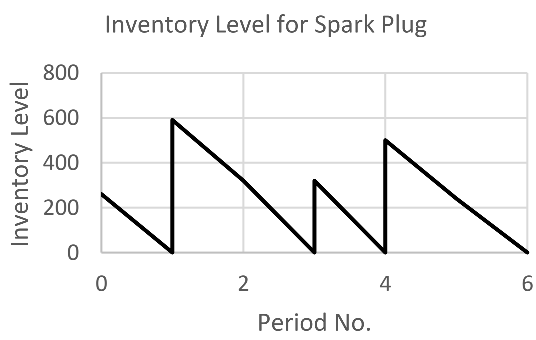

FIGURE 9: Required Quantities for Spark Plug per Purchase Order No.

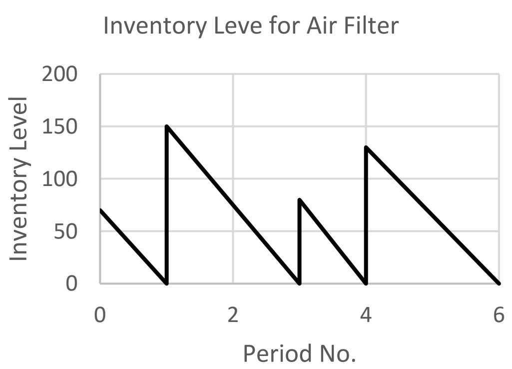

FIGURE 10: Required Quantities for Air Filter per Purchase Order No.

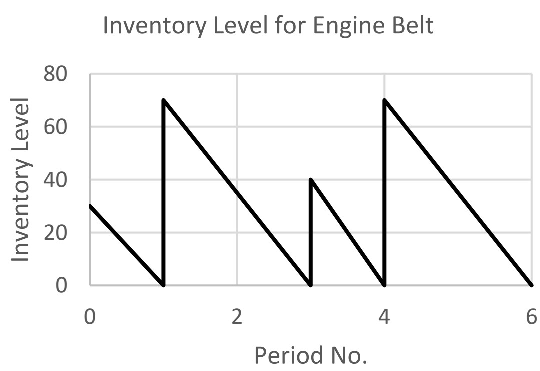

FIGURE 11: Required Quantities for Engine Belt per Purchase Order No.

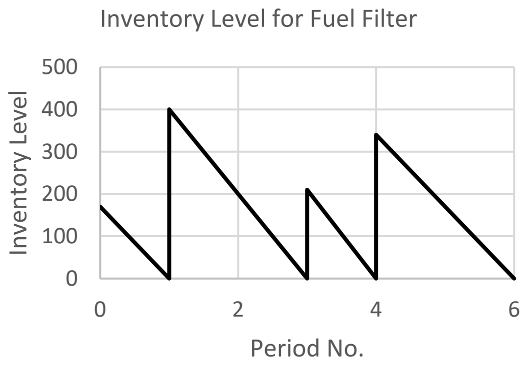

FIGURE 12: Required Quantities for Fuel Filter per Purchase Order No.

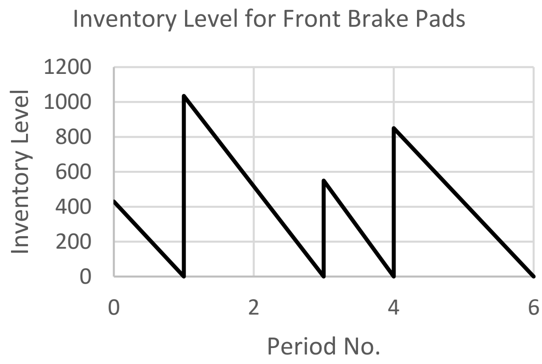

FIGURE 13: Required Quantities for Front Brake Pads per Purchase Order No.

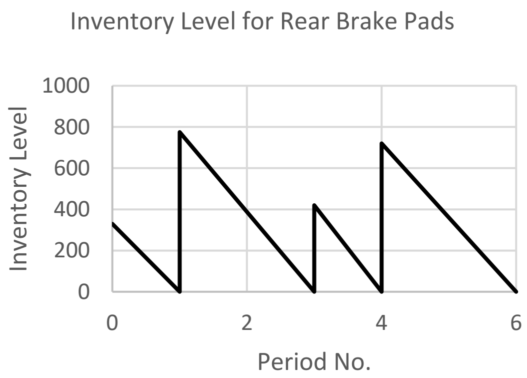

FIGURE 14: Required Quantities for Rear Brake Pads per Purchase Order No.

CONCLUSION

The assignment technique is used in its employment to propose a model that addresses the problem of inventory policy in order to achieve the minimum of the inventory cost. Taking into consideration, holding costs and ordering costs in addition to the backorder cost of the spare parts. The proposed model was applied to an automotive service center to provide a required spare parts for regular maintenance of a chosen brand model of automotive, using a back history for consuming the required spare parts over several years and calculating the average required quantities during the purchase order periods, as it was decided by the service center to make a purchase order every two months to reduce holding cost. The proposed model presents a solution that could save the total inventory costs within a year by 4% compared to current strategy for the service station.

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