

Research on Time-delay Vibration Reduction Control of 1/4 Vehicle Model

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ABSTRACT

In this paper, a 1/4 vehicle suspension model with time-delay state feedback control is established, and the maximum time-delay allowed by different feedback gains in the control system is solved according to the Lyapunov stability theory. The stability of the vehicle suspension system is analyzed, and the vibration characteristics of the system are analyzed in the frequency domain and time domain. The simulation results show that the time-delay feedback control can effectively improve the damping performance of the vehicle suspension and improve the ride comfort of vehicle.

Keywords: time-delay feedback; active suspension; time-delay stability; simulation

INTRODUCTION

In the process of driving, the vibration caused by the road roughness is transmitted to the goods and passengers through the suspension, which affects the integrity of the goods and comfort of the passengers. Therefore, how to reduce body vibration has become a hot topic for experts at home and abroad. Time delay exists in various controllable active suspensions, and the small-time delay may cause system instability. However, time delay itself has inherent damping characteristics, and time delay can be used as an active control item to reduce vehicle vibration [1].

Olgac introduced time-delay displacement feedback into the traditional dynamic vibration absorber for the first time as partial state feedback to reduce the vibration of the main system, so it is called delayed resonator [2-3]. Jalili improved on the time-delay dynamic vibration absorber and proposed a time-delay feedback shock absorber for a specific frequency range, which was implemented through a nested identification and control optimization program [4]. The research results of Zhao et al. [5] show that the time-delay nonlinear dynamic vibration absorber has a wide range of stability control and good vibration reduction effect. However, for the optimization selection of parameters, the above literature only uses the trial method to verify and select. Li and Sun took the vehicle model with dynamic vibration absorber as the research object, and discussed the vibration reduction effects of LQR control and time-delay feedback control on the vehicle model. Through comparison and simulation, they found that the two methods have a better vibration control effect. [6] With the development of time-delay dynamics, proper introduction of time-delay in the control system of automobile suspension can reduce the vibration of the car and improve the stability of the car. Therefore, the time-delay has a great influence on the performance of the vehicle system [7].

In this paper, a two-degree-of-freedom time-delay damping suspension model is established on basis of 1/4 vehicle model.

The tire time-delay state variable is used as the active suspension control parameter, and the influence of the time-delay control parameter on the suspension performance is analyzed. And verify the effect of the active suspension with time-delay feedback control in improving the ride comfort of the vehicle under the simple harmonic excitation of the road.

MECHANICAL MODEL

Simplify the vehicle according to the research purpose and establish a 1/4 vehicle model, as shown in FIGURE 1. m_e is the mass of the body; k_1 and c_1 represent the stiffness and damping coefficients of the suspension, respectively; m_c is the mass of the tire; k_2 and c_2 are the stiffness and damping coefficient of the tire; x_d is the road excitation; x_1 and x_2 represent the vertical displacement of the body and tires, respectively; u is the time-delay feedback control force.

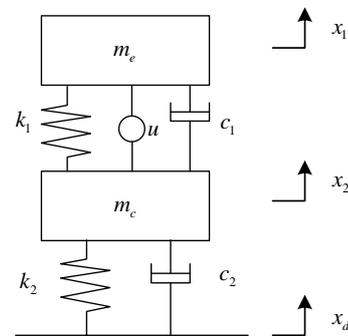


FIGURE 1: Model of 1/4 Vehicle Suspension System with Time-delay Feedback Control.

The dynamic equations of the system model are

$$\begin{aligned} m_e \ddot{x}_1 + k_1(x_1 - x_2) + c_1(\dot{x}_1 - \dot{x}_2) + u &= 0 \\ m_c \ddot{x}_2 + k_2(x_2 - x_d) + c_2(\dot{x}_2 - \dot{x}_d) - k_1(x_1 - x_2) - \\ c_1(\dot{x}_1 - \dot{x}_2) - u &= 0 \end{aligned} \quad (1)$$

Where:

$u = gx_2(t - \tau)$, g is the feedback gain coefficient, τ is the time delay.

ESTABLISHMENT OF SUSPENSION CONTROL SYSTEM MODEL

According to the performance requirements of vehicle suspension system, state variable

$X = [x_1, x^{\&1}, x_2, x^{\&2}]^T$ and output variable

$Y(t) = [x_1, x^{\&1}, x_1 - x_2, x_2 - x_d]^T$ are selected.

Then the state equation of the system is

$$\begin{cases} \dot{x}(t) = AX(t) + BU(t) + Ew(t) \\ Y(t) = CX(t) + DU(t) + Fw(t) \end{cases} \quad (2)$$

Where:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{k_1}{m_e} & \frac{c_1}{m_e} & \frac{k_1}{m_1} & \frac{c_1}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_1}{m_c} & \frac{c_1}{m_c} & \frac{k_1+k_2}{m_c} & \frac{c_1+c_2}{m_c} \end{bmatrix}, B = \begin{bmatrix} 0 \\ g \\ 0 \\ g \\ m_c \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{k_1}{m_e} & \frac{c_1}{m_e} & \frac{k_1}{m_e} & \frac{c_1}{m_e} \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, E = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{k_2}{m_c} & \frac{c_2}{m_c} \end{bmatrix},$$

$$D = \begin{bmatrix} 0 \\ -\frac{g}{m_c} \\ 0 \\ 0 \end{bmatrix}, F = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1 & 0 \end{bmatrix}, w(t) = \begin{bmatrix} x_d \\ x^{\&d} \end{bmatrix},$$

$U(t) = x_2(t - \tau)$.

SYSTEM STABILITY CONDITIONS

When the time-delay feedback control is introduced into the system, only the appropriate feedback gain coefficient can be selected to ensure the stability of the system. Therefore, it is necessary to obtain the time-delay stability interval of the system.

Laplace transform is applied to equation (2), and the characteristic equation of the system is obtained as

$$\det \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = 0 \quad (3)$$

Where:

$$a_{11} = m_e s^2 + c_1 s + k_1, a_{12} = -c_1 s - k_1 - g e^{-s\tau}, a_{21} = -c_1 s - k_1,$$

$$a_{22} = m_c s^2 + k_2 + c_2 s + k_1 + c_1 s - g e^{-s\tau}.$$

At present, the methods for analyzing the stability of time-delay systems mainly include frequency-domain methods and time-domain methods. This paper chooses the conventional Lyapunov stability theory, and judges the stability of the system through the pure imaginary roots of the characteristic equation of the system. If all the roots of the characteristic equation have negative real parts, the system is asymptotically stable. The critical condition for system instability is that the characteristic equation has pure imaginary roots $s = i\omega$.

Substitute $s = i\omega$ into equation (3), and use Euler's formula $e^{-i\omega\tau} = \cos(\omega\tau) - i \sin(\omega\tau)$ for substitution.

$$\begin{cases} \cos(\omega\tau) = \frac{k_1 m_c \omega^2 - k_1 k_2 + k_1 m_e \omega^2 + k_2 m_e \omega^2 - m_e m_c \omega^4 + c_1 c_2 \omega^2}{g m_e \omega^2} \\ \sin(\omega\tau) = -\frac{c_1 m_e \omega^3 + c_1 m_c \omega^3 + c_2 m_e \omega^3 - c_1 k_2 \omega - k_1 c_2 \omega}{g m_e \omega^2} \end{cases} \quad (4)$$

According to the trigonometric relation $\cos^2(\omega\tau) + \sin^2(\omega\tau) = 1$, the equation for ω can be obtained

$$a_4 \omega^8 + a_3 \omega^6 + a_2 \omega^4 + a_1 \omega^2 + a_0 = 0 \quad (5)$$

Where:

$$a_4 = m_c^2 + m_e^2, a_3 = c_1^2 m_c^2 + c_1^2 m_e^2 + c_2^2 m_e^2 - 2k_1 m_c m_e^2 - 2k_1 m_e m_c^2 - 2k_2 m_e m_e^2 + 2c_2 c_1 m_e^2 + 2m_c m_e c_1^2, a_2 = c_1^2 c_2^2 - 2c_1^2 k_2 m_c - 2c_1^2 k_2 m_e - 2c_2^2 k_1 m_e - g^2 m_e^2 + k_1^2 m_c^2 + 2k_1^2 m_e m_c + k_1^2 m_e^2 + 4k_1 k_2 m_e m_c + 2k_1 k_2 m_e^2 + m_e^2 k_2^2, a_1 = c_1^2 k_2^2 + c_2^2 k_1^2 - 2k_1^2 k_2 m_c - 2k_1 k_2^2 m_c - 2k_1 k_2^2 m_e - 2k_1^2 k_2 m_e, a_0 = k_1^2 k_2^2$$

The above equation defines the relationship equation $g = G(\omega)$ between g and ω , and substituting it into equation (5) can get the relationship between τ and ω .

According to the parametric equation $\begin{cases} g = G(\omega) \\ \tau = T(\omega) \end{cases}$,

the critical time delay corresponding to the feedback gain coefficient within the value range of w can be obtained, as shown in FIGURE 2.

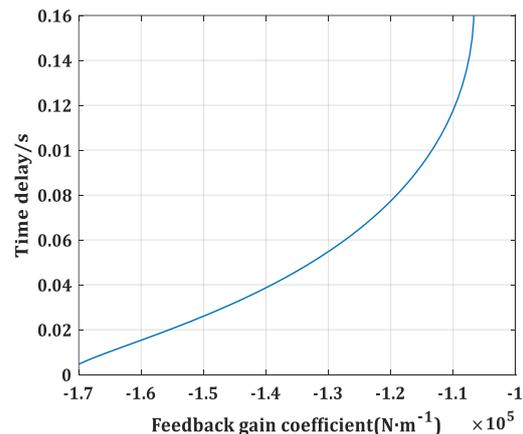


FIGURE 2: Critical Time Delay under Different Feedback Gain

The curve represents the critical time delay for system stability. Below the curve, the time-delay system is asymptotically stable. Above the curve, the time-delay system is unstable. When equation (5) has no real root, the system is stable for any time delay.

SOLVE TIME-DELAY CONTROL PARAMETERS

Fourier transformation of the dynamic equation can be obtained

$$\begin{cases} A_{11} X_e(\omega) + A_{12} X_c(\omega) = 0 \\ A_{21} X_e(\omega) + A_{22} X_c(\omega) = (k_2 + c_2 i\omega) X_d(\omega) \end{cases} \quad (6)$$

Where:

$$A_{11} = -m_e \omega^2 + c_1 i\omega + k_1, A_{12} = -c_1 i\omega - k_1 - g e^{-i\omega\tau},$$

$$A_{21} = -c_1 i\omega - k_1, A_{22} = -m_c \omega^2 + k_2 + c_2 i\omega + c_1 i\omega + k_1 - g e^{-i\omega\tau}.$$

The amplitude-frequency characteristics of the car body are calculated as

$$|H(\omega)| = \frac{|X_c(\omega)|}{|X_b(\omega)|} = \frac{|A_{12}(k_2 + c_2 i\omega)|}{|A_{12} A_{21} - A_{11} A_{22}|} \quad (7)$$

Equation (7) is only a function of the feedback gain coefficient and time delay. According to the theory of differential equations, when the partial derivatives of equation (7) to g and τ are zero, the function takes the extreme value [8].

$$\frac{\partial |H(\omega)|}{\partial g} = \frac{\partial |H(\omega)|}{\partial \tau} = 0 \tag{8}$$

Since equation (8) contains a transcendental function term, which does not satisfy any function of a polynomial equation with a polynomial as a coefficient, the equation will have infinitely many solutions. The corresponding $|H(\omega)|$ has an infinite number of extreme points.

According to the stability constraints, select the minimum points in the time-delay independent stability zone, and the corresponding ones are the optimal feedback parameters to be sought. And the minimum point close to the critical line of the stable region has a better damping effect.

Draw the curves of $\frac{\partial |H(\omega)|}{\partial g} = 0$ and $\frac{\partial |H(\omega)|}{\partial \tau} = 0$

respectively, as shown in FIGURE 3. The parameters of the vehicle model are shown in TABLE 1.

In the FIGURE (3), the solid line represents the extreme point distribution of the time delay value, while the dotted line represents the extreme point distribution of the feedback gain coefficient. The intersection point of the two curves is the solution of the system, and there are infinitely many periodic distribution solutions to the equation. Combined with the stability conditions, the optimal feedback gain coefficient and time delay value can be obtained

$$g = -22670N \cdot m^{-1}, \tau = 0.2419s$$

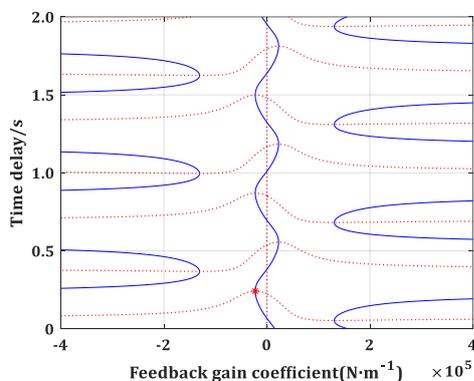


FIGURE 3: Extreme Point Distribution

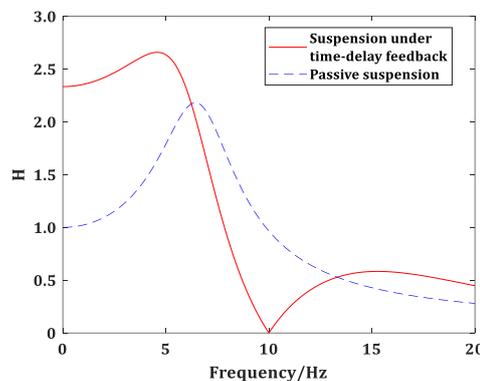
TABLE 1: Simplified Vehicle Parameters

Vehicle parameters	Value	Vehicle parameters	Value
m_e / kg	345	$k_2 / (N \cdot m^{-1})$	192000
m_c / kg	40.5	$c_1 / (N \cdot s \cdot m^{-1})$	1500
$k_1 / (N \cdot m^{-1})$	17000	$c_2 / (N \cdot s \cdot m^{-1})$	0

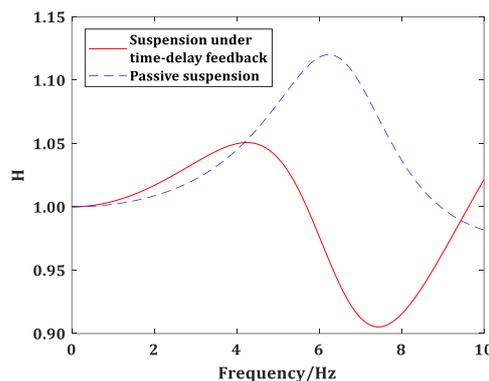
SIMULATION ANALYSIS

Frequency Domain Analysis

The optimal time-delay feedback control parameters are introduced into equation (3), and the amplitude-frequency characteristic diagram based on the body displacement is obtained, as shown in FIGURE 4.



(a) Vehicle Body



(b) Wheel

FIGURE 4: Amplitude Frequency Characteristic Diagram

It can be seen from FIGURE 4 (a) that under the time-delay feedback control, the body amplitude is greatly attenuated within 6.2Hz~13.4Hz, and the vibration is completely suppressed at 10Hz. It can be clearly seen from FIGURE 4 (b) that the time-delay feedback control can significantly improve the amplitude-frequency characteristics of the tire, especially in the low frequency range

Time Domain Analysis

Time-domain simulation of system vibration response was carried out in the MATLAB/Simulink module. $x_d = 0.05 \times \sin(10t)$ with harmonic excitation was taken as the input of road vibration, and the optimal time-delay control parameters were substituted into the system. The simulation figure is shown in FIGURE 5.

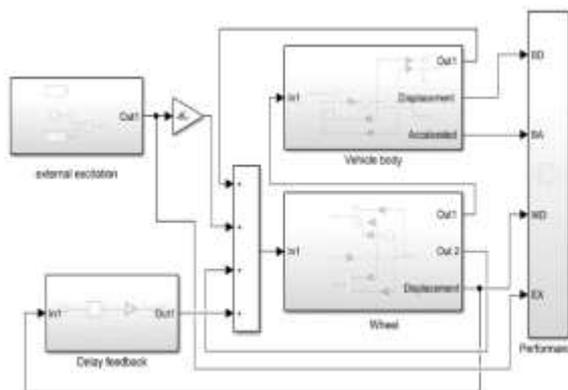


FIGURE 5: Simulation Model

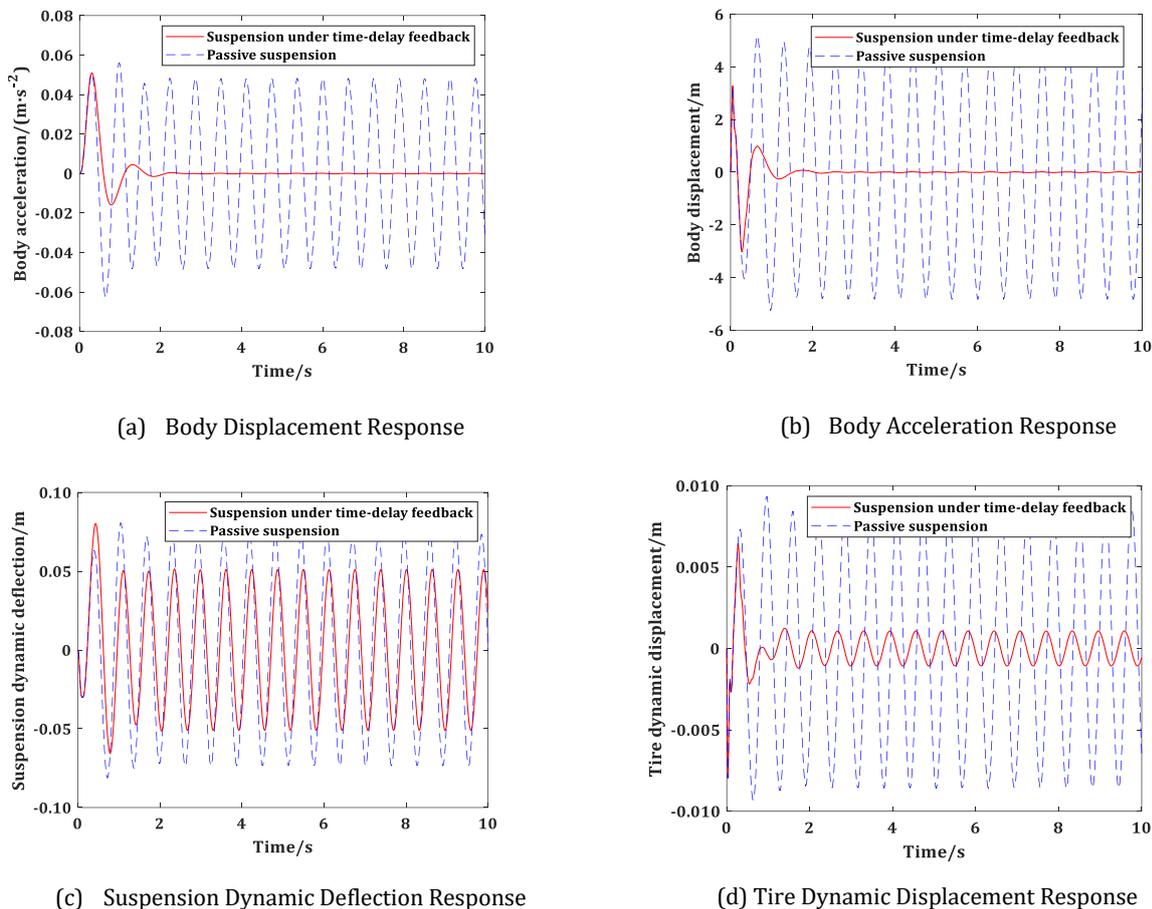


FIGURE 6: Simulation Comparison Under Simple Harmonic Excitation

It can be seen from FIGURE 6 that compared to passive suspension, when the road excitation frequency is close to the natural frequency of the vehicle body, the body displacement and body acceleration are significantly reduced, and the suspension dynamic deflection and tire

dynamic displacement are also reduced to varying degrees. According to the root mean square (RMS) value of each vehicle performance in TABLE 2, under the time-delay feedback control, the RMS of body displacement in FIGURE 4 (a) drops

TABLE 2: RMS Value of Vehicle Performance Indicators

performance index	passive suspension	Suspension under time-delay feedback	Reduction ratio
RMS of body displacement m	0.0333	0.0082	75.39%
RMS of body acceleration $m \cdot s^{-2}$	3.2682	0.4669	85.71%
RMS of suspension dynamic deflection m	0.0513	0.0381	25.83%
RMS of tire dynamic displacement m	0.0058	0.0012	79.10%

from 0.0333 to 0.0082, and the reduction ratio is 75.39%. The RMS of body acceleration in FIGURE 4 (b) drops from 3.2682 to 0.4669, and the reduction ratio is 85.71%. The RMS of suspension dynamic deflection in FIGURE 4 (c) drops from 0.0513 to 0.0381, and the reduction ratio is 25.83%. The RMS of dynamic displacement in FIGURE 4 (d) drops from 0.0058 to 0.0012, and the reduction ratio is 79.10%.

From the above analysis, it can be concluded that the time-delay feedback control can effectively improve the damping performance of the suspension system and the grounding performance of the tires, and improve the ride comfort of the vehicle.

CONCLUSIONS

In this paper, time-delay state feedback is introduced into the 1/4 vehicle model to conduct stability analysis and simulation of the time-delay control system, and the following conclusions are drawn:

- (1) The time-delay feedback control can effectively reduce the acceleration of the vehicle body and improve the vibration damping performance of the vehicle suspension. At the same time, tire vibration has not deteriorated, improving the ride comfort of the vehicle.
- (2) In the time-delay feedback control, if the control parameters are not selected properly, the system may be unstable and the vibration reduction effect may become worse.

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