International Journal of Scientific Advances
ISSN: 2708-7972
Volume: 3 | Issue: 2 | Mar - Apr 2022 Available Online: www.ijscia.com
DOI: 10.51542/ijscia.v3i2.1

Detailed Study of Stellar Evolution and Stellar Gravitational Collapse Leading to The Formation of Black Holes

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ABSTRACT

In this paper we talk about the formation mechanism of the stellar gravitational singularities. To begin with we talk about the formation and evolution of stars and understand the Hertzsprung Russell diagram that teaches us how to classify stars. With the help of the diagram, we categorize the stars based of their physical parameters such as color, temperature, and mass. We then talk about the death cycle of different mass stars and what comes after when their fuels have been exhausted. Stars below the Chandrashekhar limit form a white dwarf at the end of their lives, while stars above the limit form a neutron star of a singularity. Further to find which of the heavy mass stars forms a singularity we look at the Tolman-Oppenheimer-Volkoff limit that states stars with mass above the limit will form singularities. The types of singularities formed depend on the solution of the general theory of relativity given by Schwarzschild, Kerr, Kerr-Newman, and Reissner-Nordström. Each of the theory aspect of the four solutions has been described to give a better understanding of the structure of the singularity formed. The paper also explains theories such as Wormholes and time travel in brief to try and explain what can replace the singularity.

Keywords: Hertzsprung Russell; Chandrashekhar Limit; White Dwarf; Neutron Star; Singularity; Tolman-Oppenheimer-Volkoff Limit; Schwarzschild; Kerr; Kerr-Newman; Reissner-Norström; Wormhole; Time travel

INTRODUCTION

In this paper we have summarized the stages in the life cycle of a star right from the birth till the death. Our main focus in this paper is to make everyone understand about the life of star and how a star evolves to form a black hole. We have explained about the conditions required for the formation of the singularity, different types of singularity, and hypotheses involved in the process.

STAR FORMATION

Stars are born within molecular clouds, clouds of gas and dust, and scattered throughout galaxies. Mainly composing of hydrogen, turbulence deep within these clouds gives rise to blobs with sufficient mass that the gas and dust begin to collapse under its own gravitational attraction. As the cloud collapses, there is increase in gravitational attraction, which further accelerates the collapse. The material at the center begins to heat up. This cannot be halted until large turbulent motions have been created inside the collapsing blob, providing enough thermal pressure to balance gravity. Known as a protos tar, the hot core at the heart of the collapsing cloud, will one day become a star [1].

An interstellar cloud of gas will remain in hydrostatic equilibrium as long as the kinetic energy of the gas pressure is in balance with the potential energy of the internal gravitational force. Mathematically this is expressed using the virial theorem, which states that, to maintain equilibrium, the gravitational potential energy must equal twice the internal thermal energy [2]. If a pocket of gas is massive enough that the gas pressure is insufficient to support it, the cloud will undergo gravitational collapse. The mass above which a cloud will undergo such collapse is called the Jeans mass. This mass depends on the temperature and density of the cloud but is typically thousands to tens of thousands of solar masses [3].

WHAT TRIGGERS THE FORMATION?

First, the interplay between the interstellar radiation field and molecular self-shielding determines what fraction of the gas is in molecular form and thus eligible to form stars. Second, internal feedback determines the properties of the molecular clouds that form, which are nearly independent of galaxy properties until the galactic interstellar medium (ISM) pressure becomes comparable to the internal giant molecular cloud (GMC) pressure. Above this limit, galactic ISM pressure determines molecular gas properties. Third, the turbulence driven by feedback processes in GMCs makes star formation slow, allowing a small fraction of the gas to be converted to stars per free-fall time within the molecular clouds [4].

A variety of processes cause interstellar gas to become cold enough and dense enough to form stars. On galactic scales, stellar instabilities, spiral waves, and global perturbations like bars can move the gas around supersonically and cause shocks to form that are larger than the characteristic size of a gravitational instability in the gas. Then giant cloud complexes form from the ambient gas. As these complexes dissipate their internal turbulent energy, they contract gravitationally and fragment because of converging and diverging turbulent motions until dense, thermally dominated cold cloud cores form. Stellar pressures also compress the gas supersonically. On sufficiently large scales, these compressions lead to collapse in shells and rings. On small scales, stellar pressures can turn in pre-existing clumps unstable to collapse, especially along the edges of HII regions and super-bubbles.

The empirical laws of star formation have no obvious connection to the details of these triggering mechanisms. The empirical laws state mostly that star formation requires cold and dense gas. The non-linear dependence of star formation rate on total gas density and empirical evidence suggests also that general dynamical processes in the ISM are involved in determining the time scale. Since triggering on the length scale of these laws has the same dynamical time scale, all of the various triggering processes can mix together without much distinction. The distinct contribution that triggering makes to the star formation rate might be most evident in large-scale regions where the average molecular fraction is neither very high nor very low. There the dynamical processes related to cloud formation could have a significant influence on the abundance of cold gas in clouds. There might still be a linear relation between cold gas mass, and star formation rate at these places because star formation follows cold gas no matter what forms the cold gas, but the rate of both cold gas formation and star formation could be modulated by dynamical processes more there than elsewhere [5].

STELLAR EVOLUTION

Hertzsprung-Russell Diagram:

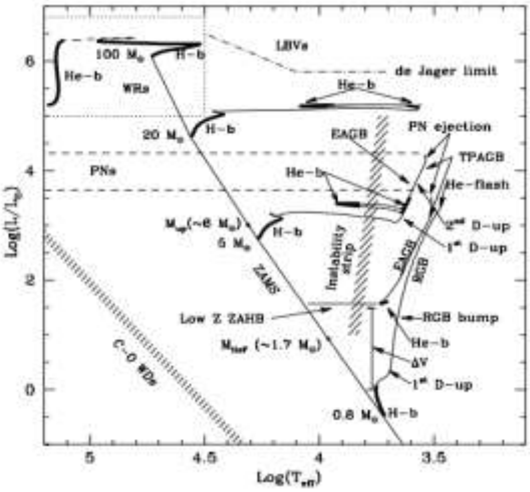

FIGURE 1: Hr Diagram Showing Evolutionary Paths of Different Mass Stars.

Fig 1. Shows the evolutionary paths in the HRD of model stars of composition [Z=0.008, Y=0.25] and of initial mass 0.8 M^{-}, 5 M^{-}, 20 M^{-}, and 100 M^{-}. The models are calculated with the overshoot scheme for central convection. MHeF and Mup are the masses separating low-mass stars from intermediate-mass stars, and the latter from the massive ones, respectively. For low and intermediate-mass stars the tracks go from the zero-age main sequence (ZAMS) to the end of the asymptotic giant branch (AGB) phase, whereas for the massive stars they reach the stage of C-ignition in the core. Massive stars include the effect of mass loss by stellar wind. H-b and He-b stand for core H- and He-burning, respectively. He-flash indicates the stage of violent ignition of central He-burning in low-mass stars at the tip of the red giant branch (RGB). The main episodes of external mixing (1st and 2nd dredge-up) are indicated by 1st D-up and 2nd D-up, respectively. The AGB phase is separated into early stages (E-AGB) and thermally pulsing regime (TP-AGB) of the He-burning shell. For low- and intermediate-mass stars we show the stage of planetary nebula (PN) ejection, the region where PN stars are observed, and the white dwarf (WD) cooling sequence. The horizontal line labelled ZAHB indicates the locus of the zero-age horizontal branch – core He-burning models – of low-mass stars with composition typical of globular clusters. The shaded vertical band shows the instability strip of Cepheid and RR Lyrae stars. In the region of massive stars, we show the de Jager limit, the location of the blue luminous variables (LBVs) and Wolf-Rayet stars (WRs). Finally, the thick portions of the tracks indicate the stages of slow evolution, where the majority of stars are observed.[7]

Pre-Main Sequence Stars: Once the star is formed then its radiation pressure sweeps away the residual gas of the original cloud. The star is now in a Pre-Main Sequence (PMS) phase. At the end of proto-stellar phase a star is formed which is in perfect equilibrium condition where the stellar gas force counterbalances the gravitational force. However, the star is warm and radiates, loosing energy from the surface; thus, after a while, the star cools down and its pressure decreases becoming insufficient to completely sustain the star against the gravity. The star slightly contracts increasing its temperature and reaching again the hydrostatic equilibrium until the radiation losses from the surface cools down again the star. The PMS phase is characterized by these consecutive stellar contractions when the star passes from one “quasi - equilibrium” state to another one, while its internal temperature increases. During the PMS evolution a star reaches first the temperature for deuterium and light elements (Li, Be, B) burning, then it reaches the temperature for hydrogen burning into helium in its core. The hydrogen fusion into helium is the first nuclear reaction able to produce an energy which can replace the radiative losses from the surface and counterbalance gravitational force hence attaining Hydrostatic Equilibrium.[1]

Main Sequence Stars: Now with the initiation of hydrogen fusion, star enters the Main Sequence Phase. We know that inside the core the nucleosynthesis occurs through PP – Chain reactions in which H is converted to He. The temperature and pressure conditions required for these reactions to occur is satisfied by only some part of the core. These conditions are decided by the mass of the star. When the H in this region is all fused up and He is formed then the start has a He core and large part of the star is H. The nuclear energy generation is stopped, hence there is no force to counteract the gravity hence the core starts collapsing. Now this contraction happens differently with different mass stars.

Post Main Sequence: With the core the hydrogen layers surrounding the core also start contracting. Due to this contraction the temperature and pressure in these surrounding layers increase and reach a level which satisfy the conditions required for the fusion of hydrogen. Now hydrogen starts burning in a shell around the core which is now the source of energy required to counteract the gravitational collapse. As more hydrogen burns more helium is added into the core which again causes further gravitational collapse. This gravitational collapse pulls the edges of the hydrogen burning shell towards the core and to counteract this gravitational pull more and more hydrogen starts burning and more energy is produced. The amount of hydrogen burning in the shell is much more as compared to that in the core hence a very large amount of energy is produced in this stage as compared to main sequence stage. All of this is happening near the core, but the observational effects are at the surface in the form of luminosity. Hence this energy needs to be transferred to the surface in order to be observed. So, as this energy is transferred towards the surface the star starts to expand, more energy generated within the shell more the star expands. During this process the surface temperature drops, but it cannot drop under certain value. At this point the energy is transferred through convection rather than radiation because convection energy transfer is more efficient to transfer heat to the surface. Due to the turbulence the layers which are deep come to the surface that is, the mixing of the layers occurs hence the composition of star in the outer layers is different from that near the core. This phase is known as Red Giant phase, and it lasts for about a billion years.

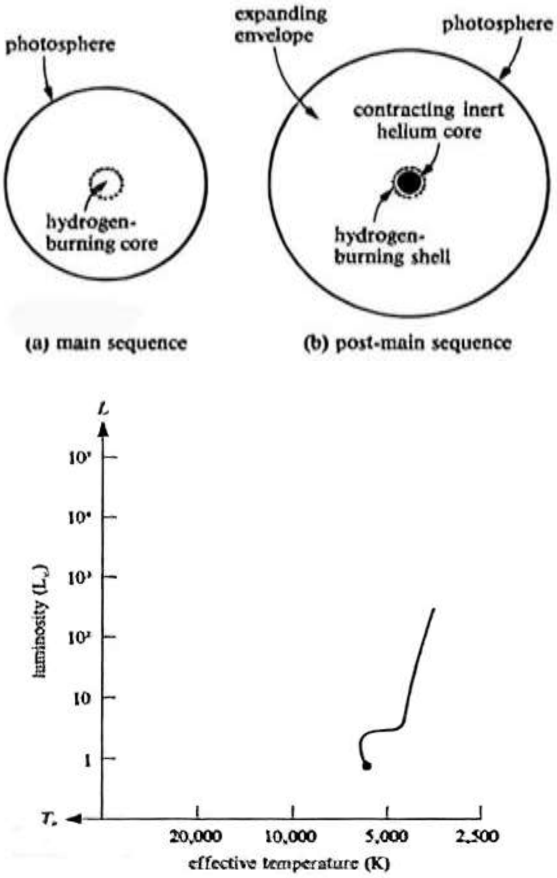

FIGURE 2: The Structure of a Star (A) In Main Sequence Phase, (B) As It Begins to Leave the Main Sequence Because Of Core Hydrogen Exhaustion And (C) Ascent of A Low Mass Star to Red-Giant Branch.

There are mainly three different types of stars based on the mass. When they leave the main sequence all of them experience a very different fate as compared to others.

1. Low Mass Stars: By low mass stars we define those which shortly after leaving the main sequence toward the red giant branch (RGB), develop an electron degenerate core composed of helium. When the mass (MHe) of the He core has grown to a critical value (0.45 - 0.50 MSun), the precise value depends on the composition, star mass, and input physics), a He-burning runaway is initiated in the core (He-flash), which continues until electron degeneracy is removed. The maximum initial mass of the star (otherwise called MHeF) for this to occur is about 1.8 - 2.2MSun, depending on the initial chemical composition. Within the same mass range, we distinguish the stars lighter than Mcon ≈ 1.2 - 1.3MSun that burn hydrogen in a radiative core from the more massive ones doing it in a convective core.[7] The core H-burning main sequence phase of stars lighter than Mcon is characterized by the gradual formation of a small He core at the center. After the main sequence phase, the H exhausted core temporarily cools as electron degeneracy sets in, and the energy liberated by gravitational contraction flows out by electron conduction, delaying the increase in central temperature required to ignite helium in the core. As a low-mass star reaches the base of the RGB, the central temperature reaches a minimum approximately equal to the temperature of the H-burning shell. Thereafter, the mass of the helium core grows under the action of the H burning shell, the core contracts, and temperatures in the core and H-burning shell increase.[7] Stars with masses lower than about 2.3 MSun, whose interiors did not reach very high temperatures, remain long time in the RGB phase because the reaching of the He burning temperature is more difficult.[1]

2. Medium Mass Stars: Stars more massive than MHeF are classified either as intermediate-mass or massive stars. In turn we distinguish the intermediate-mass stars from the massive ones by looking at the stage of carbon ignition in the core. By intermediate mass we mean those stars which, following core He-exhaustion, develop a highly degenerate carbon-oxygen (C-O) core, and as asymptotic giant branch (AGB) stars experience helium shell flashes or thermal pulses. The AGB phase is terminated either by envelope ejection and formation of a white dwarf (MHeF ≤ Mi ≤ Mw) or carbon ignition and deflagration in a highly degenerate core once it has grown to the Chandrasekhar limit of 1.4 MSun.[7] Stars which ignite He in the center but did not explode as supernovae end their life as white dwarfs with a C/O core or with a Ne core if they succeed to ignite carbon too.[1]

3. High Mass Stars: Finally, massive stars are those that ignite carbon non-violently and through a series of nuclear burnings proceed either to the construction of an iron core and subsequent photodissociation instability with core collapse and supernova explosion (Mi ≥ Mmas), or following a more complicated scheme undergo core collapse and supernova explosion (Mup ≤ Mi ≤ Mmas). Mmas is about 12 MSun.[7] In intermediate and high-mass stars, the main sequence core H-burning phase is characterized by the formation of a convective core. The main sequence core H-burning lifetime goes from several 108 years to a few 106 years as the mass of the star increases from about 2 to 100 MSun. [7] After the central H burning the stars experience a phase in which H is burned in a shell around a central inert, He core. In this phase the stellar envelope expands and cools down and the star moves toward the red in the HR diagram (Sub Giant Branch). during the Sub Giant Branch H burning mainly occurs through the CN-NO bi-cycle, much more sensitive to the temperature than the concurrent proton-proton chain. When CNO hydrogen burning reaches the equilibrium of secondary elements in the cycle, the stars enter in the Red Giant Branch.[7]

Degeneracy in White Dwarfs Stars and Neutron Stars:

Electron degeneracy in white dwarfs

The white dwarf star is a small, dense star with a mass less than the Chandrasekhar mass limit of 1.44 solar masses. Many white dwarf stars are born out of red giant stars that have burned their fuel and lost a large fraction of their mass. These stars have fused all of their hydrogen and helium, leaving an abundance of carbon and, in many cases, oxygen in the core. At this point, the star no longer has enough energy to fuse carbon into a heavier element so the white dwarf star contracts. This contraction compresses the electrons in the core into degenerate energy levels, forming the electron degeneracy pressure. A typical white dwarf with a mass less 3 than 1.44 solar masses will become stable after balancing the gravitational pressure due to the contraction, and the electron degeneracy pressure.[8]

Neutron degeneracy in neutron stars

More massive stars, typically greater than eight times the mass of our sun, however, are energetic enough to fuse elements up to iron. Fusion can no longer naturally occur in stars past iron. When iron is formed, it is deposited in the core, causing the density to rapidly increase and the core to begin to contract inward. This causes temperatures to rise in the core to help resist collapse. The rise in temperature and density allows for electron capture in the core by the reaction,

$$p^{+} + e^{-} \leftrightarrow n + \nu_{e}$$

Both neutrinos and neutron rich matter are produced at the core of these large stars. Eventually, the core of these larger stars will become too massive, causing a gravitational core collapse supernova which, in many cases, leave behind a neutron star. These neutron stars are neutron rich due to the above reaction. Neutron stars are much denser than white dwarf stars, which, once again, causes the core of the stars to collapse. The compression of neutrons in the contracting core, however, creates a neutron degeneracy pressure. This pressure counteracts the gravitational collapse of the star. [8]

FORMATION OF SINGULARITIES

Years of scientific advancements has brought us the widely accepted theory that when a massive star collapses under the force of its own gravity, the final fate of such a continual gravitational collapse will be either a black hole or a naked singularity within the framework of general theory of relativity [9]. Computations suggest collapse processes of the order of tens of milliseconds. The release of gravitational binding energy in supernovae is typically of the order of 10^{51} ergs and in the more extreme case of neutron star collapse, perhaps as high as 10^{54} ergs [10]. What determines fully the final fate of collapse here are the initial density and velocity profiles for the collapsing shells of matter [9]. Core collapse events certainly produce gravitational waves: large amounts of mass ($\sim 1–100 M_{\odot}$) flow in a compact region ($\sim 10^{8}$ –10^{9} cm) at relativistic velocities ($v/c \sim 1/5$). These characteristics are necessary conditions for a source to be an interesting GW source [11]. Once a star starts collapsing, it reaches various temporary equilibrium states. But none of them can halt the collapse forever. Once the collapse had begun, whether it can stop or continue without stopping depends on whether it reaches a critical dimension known as gravitational radius. Collapse can be halted anywhere outside but never inside the radius. [12]

There are 3 critical stages based on this.

1. First Critical Stage:

At this stage, the star contracts to 1.5 times the gravitational radius. At this stage, all photons emitted from the surface escape into space and could be seen by a distant observer. Photons emitted at a tangent to the star's surface are caught in spherical cloud from which they slowly leak forever.

2. Second Critical Stage:

At this stage, the star has contracted to its gravitational radius. Now only those photons that leave perpendicularly to the surface can escape and they form a second leaking clouds just outside the gravitational radius.

3. Third Critical Stage:

At this stage, the star has contracted beyond gravitational radius. Now it collapses endlessly into a singularity, point of infinite density from which nothing can escape.

The matter and energy densities, spacetime curvatures, and all physical quantities blow up and take extreme values in the limit of approach to a spacetime singularity [13], housed with an event horizon or otherwise. Despite the fact that considerable work has been done in past years to understand the dynamical gravitational collapse in general relativity, the nature of cosmic censorship, black holes and naked singularities remain one of the most important unresolved issues in gravitation theory and black hole physics today [9]. It is expected that the successful combination of quantum mechanics and gravity will lead to the regularization of these singularities [14]. Following models are widely accepted to describe the phenomena of formation of singularities:

1. The Oppenheimer-Snyder-Datt model: The model assumes the density to be homogeneous, symmetry to be spherical and neglects gas pressure [13]. The calculations showed that an event horizon develops as collapse progresses, such that no material particles or photons from the region escape too far away observers. Once the star collapses to a radius smaller than the horizon, it enters the black hole, finally collapsing to a spacetime singularity with extreme densities that is hidden inside the black hole and invisible to any external observers. For the collapsing star to create a black hole, an event horizon must develop prior to the time of the final singularity formation [9] In the OSD case, all matter falls into the spacetime singularity at the same comoving time, while the event horizon forms earlier than the singularity, thus covering it.

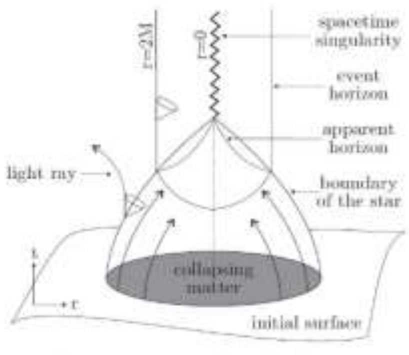

FIGURE 3: Dynamical Evolution of a Homogeneous Spherical Dust Cloud Collapse, As Described by The Oppenheimer-Snyder-Datt Solution.

2. Cosmic Censorship Conjecture.: While the general theory of relativity necessarily implies the formation of a spacetime singularity as the end state for a massive collapsing star, such a singularity will always be necessarily hidden within a black hole. Such an assumption is known as the cosmic censorship hypothesis despite much work in studying the censorship and its implications, the issue of final fate of a complete gravitational collapse of a massive star remains far from being fully resolved [13].

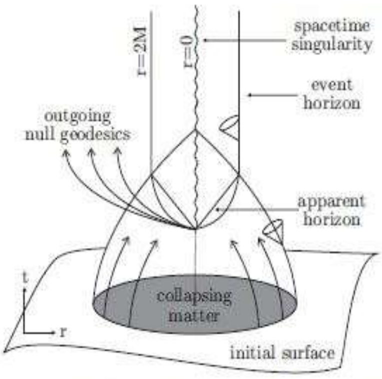

FIGURE 4: A Spacetime Singularity of Gravitational Collapse Which Is Visible to External Observers in The Universe, In Violation to The Cosmic Censorship Conjecture.

Charged stars have the potential of becoming charged black holes or even naked singularities. Several features could be common to the evolution of rotating collapsing stars, with the angular momentum playing the role of the electric charge [15]. But there exists a general consensus that astrophysical objects with large amounts of charge cannot exist in nature [15]. Considering inhomogeneities in the initial density profile it is possible to show that the behavior of the horizon can in fact change drastically, thus leaving two different kinds of outcomes as the possible result of generic dust collapse: the black hole, in which the horizon forms at a time anteceding the singularity, and the naked singularity, in which the horizon is delayed

Subsequently, researchers found many scenarios of inhomogeneous pressure-less collapse where strong-curvature naked singularities, the ones that are genuine crushing singularities, developed from regular initial conditions. A general treatment for such a scenario was developed by Joshi and Dwivedi in 1993. In particular, it became clear that while the homogeneous pressure-less collapse considered by Oppenheimer and Snyder produced a black hole, a more realistic density profile with density higher at the center and decreasing as one moves away can give rise to a naked singularity, which is an intriguing situation indeed. While these dust collapse models ignored pressure, the general techniques developed in the above work to understand the dynamical evolution of collapse did find applications later when collapse models with pressure were considered [9].

Indeed, while it is by now certain that the outcome of a realistic classical collapse is necessarily a standard black hole delimited by an event horizon , it has recently been suggested that only apparent or trapping horizons might actually be allowed in nature, and that somehow semiclassical or quantum gravitational effects could prevent the formation of a (strict, absolute) event horizon [16] and hence possibly evade the necessity of a singular structure Note that Hawking radiation would still be present, even in the absence of an event horizon.

STRUCTURE OF SINGULARITY

We will start 'Schwarzschild Black Hole', instead of looking at it as the solution to Einstein's field equations, we will examine its physical origin which is the end product of a collapse of a massive star. However, a spinning black hole cannot be described using this and so we will talk about the 'Kerr Solution'. This would help in understanding how a real black hole works.

A. Schwarzschild Black Hole:

We have two kinds of black holes one is formed due to the supernova explosion and has mass of 3-10 solar masses and the second is the supermassive black holes that are formed at the center of the galaxy and have masses up to billions of solar masses.

Massive stars die only due to 3 reasons:

1. the fusion stops with Fe nuclei
2. at these high temperatures the photons produced

by thermonuclear reactions start the cause the heavy nuclei to disintegrate to smaller nuclei and all the energy that was liberated by the fusion has to be paid back.

3. high temperatures allow reactions in which neutrinos are produced, they barely interact with ordinary matter, and so stream out of the central core, carrying large amount of energy with them.

The final result is that in the space of a just a few seconds, the entire central core suddenly finds itself with no fuel and a sudden drop in temperature. Only one thing can then happen - the core collapses, and the outer layers of the star then begin to collapse inwards as well, since there is no longer anything holding them up. All the nuclei have been crushed together, and at the end of this precipitous collapse we end up with a dense fluid of neutrons, with the protons and electrons forced together to form neutral matter. But the collapse then stops - in fact, the quantum degeneracy pressure of the neutrons themselves finally stops the inward rush, and there is then a huge 'rebound' effect: the inner core explodes back outwards, with a massive shockwave propagating out to meet the outer parts of the star, which are still falling inwards. This is the beginning of what we call a supernova. [20,25] This is where we can finally find the connection to black holes. For using similar ideas to those of Chandrasekhar, but this time incorporating General Relativity, physicists beginning with Oppenheimer realized that there were only 2 possibilities available to the supernova remnant. By far the most likely is that it will settle down to form a neutron star, which is initially spinning very rapidly, but which soon slows its rotation, and gradually cools down.[20]

Gravitational/Stellar collapse occurs when stars internal pressure is not enough to resist the gravity of the star. This normally occurs because the star has less to no fuel left to maintain its temperature through stellar nucleosynthesis or if a stable received mass in a way that would not be useful in increasing the temperature. In both the cases the temperature of the star is not enough to stop it from crushing under its own weight.

The collapse can be stopped by degeneracy pressure of the star's constituents, allowing the condensation of matter into an exotic denser state [18]. It forms various types of compact stars like white dwarfs, neutron stars etc. If the mass remnant exceeds 3-4 solar masses (Tolman-Oppenheimer-Volkoff limit) either because the original star was very heavy or because the remnant collected additional mass through accretion of matter, even the degeneracy pressure of neutrons is insufficient to stop the collapse. No known mechanism is powerful enough to stop the implosion and the object will inevitably collapse to form a black hole. [18,19] Near the surface of the star the gravitational field becomes stronger, and the light begins to bend back towards the star. Finally, an event horizon is formed, the gravitational field below the event horizon is said to be so strong that light or any other matter cannot escape once it enters the event horizon. Inside this event horizon a spacetime singularity forms and everything that fall into the event horizon has to fall to the singularity. Once formed a black hole keeps eating to grow in size. For a symmetric black hole all the matter directly falls inwards to the center. This spherically symmetric black hole is called a, “Schwarzschild black hole”, since Schwarzschild’s 1916 solution of Einstein’s equations actually describes this system. It is the simplest kind of black hole.[20]

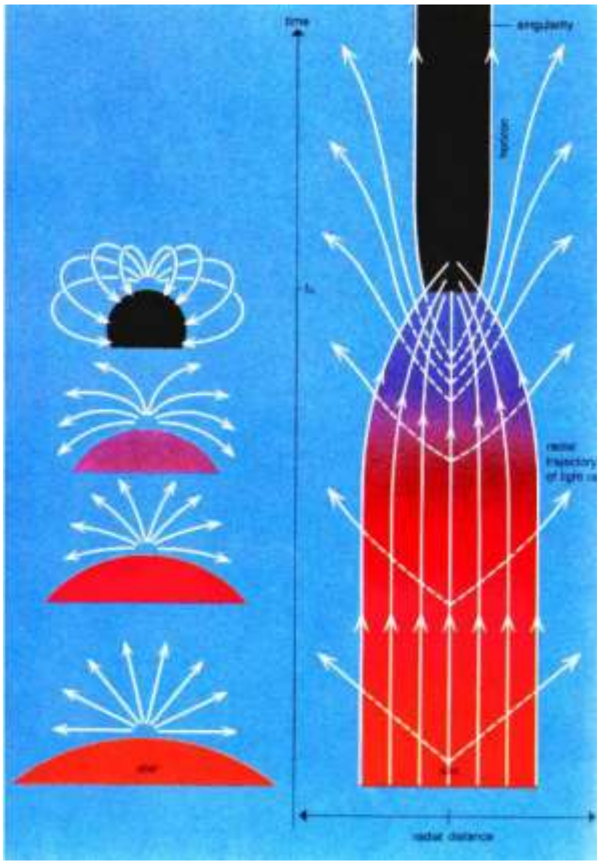

FIGURE 5: Formation of Schwarzschild Black Hole [20]

The next question would be how big is a black hole? The radius of these black holes is called the Schwarzschild radius and is directly proportional to the mass inside the event horizon. The result of the Schwarzschild radius found by using the equation:

$$r_s = \frac{2GM}{c^2}$$

Where

r_s is the Schwarzschild radius

G is the gravitational constant i.e., 6.674×10^{-11} $m^3 \cdot kg^{-1} \cdot s^{-2}$

M is object mass

c is the speed of light

So, for a star as big as the sun the black hole formed would be about 2.9 kms wide. Whereas for a star the size of Earth will have a radius of approx. 9mm. This radius means that the mass of the entire star has been collapsed into a diameter of approx. 18mm.

By looking at the Schwarzschild solution for the first time it is obvious to anyone that a collapse is a very messy situation, even if the process starts of symmetrically. There can be instabilities during the collapse but if we show that the initial collapsing material do not have any net angular momentum, then irrespective of the disorganized collapse the end result will always be a symmetric black hole. Even if the event horizon formed is not in symmetric state, it will radiate away gravitational waves and other stuff to form a symmetric black hole. [16,20]

A1. The wormhole theory

The Schwarzschild solution can be extended even further, in 1935 Einstein and Rosen used GR to elaborate by proposing the existence of "bridges" through space-time. These bridges can connect two different points in spacetime, theoretically creating a shortcut which would reduce space travel time and distance. The shortcuts came to be called Einstein-Rosen bridges, or wormholes.

Wormholes contain two mouths, with a throat connecting the two, the mouths would most likely be spheroidal. The throat might be a straight stretch, but it could also wind around, taking a longer path than a more conventiona l route might require. The theory of GR predicts the existence of them, but none have been found so far. [20,21,22] These wormholes might not only connect two different locations in the universe, but they may also connect two separate universes with each other. If the mouth of the wormhole moves in a specific manner, then the wormhole can allow for time travel. If there is a region of ‘negative mass-energy’ in the bridge, it could open up for a brief time and form a wormhole. The possibility of joining two different spacetime could be done mathematically however physically it gave no result. Without the negative mass-energy density all we have is a black hole singularity embedded in two different spacetimes. [21,22]

Hawking realized that a negative energy density can be developed in the area of a singularity, if black hole evaporation via the Hawking radiation (the black hole can radiate and loss mass) takes place. The idea is that the apparent loss of information vis hawking radiation is actually recovered in an alternate world present in the interior of the black hole inaccessible to our world. If this solution is actually true, then the singularity present in the center of the black hole will be replaced by a wormhole. [20,26]

However, these are speculations and will not discussed too much into the depth as it is beyond the scope of this paper.

B. Kerr solution

We know that for a star to collapse into a black hole without having any spin is rather difficult to achieve. The collapse of a star would make the star spin even faster than before as it gets smaller and the end rest of that would be a rapidly spinning black hole. So how is this model different than a Schwarzschild model? Well to start with the black formed with the Kerr solution will not have a simple event horizon but two different horizons and the arear between these two new horizons would be refer to as the ergosphere. The outer horizon is called the rotational horizon, it is used to identify the surface below that cannot stop rotating no matter the force applied to it.

An object that falls into the ergosphere can still escape provided it does not fall as far as the inner horizon that acts exactly like an event horizon of a Schwarzschild black hole- nothing can escape the event horizon. Another way to think about the Kerr black hole is like the twist in spacetime. Any object moving in this region would be affected and will have a change in the motion accordingly. So, after crossing the inner horizon of Kerr black hole what happen next? The structure of the singularity of the Kerr black hole is very different from before. It is a ring i.e., spacetime develops an infinite curvature at all points on this singular ring.

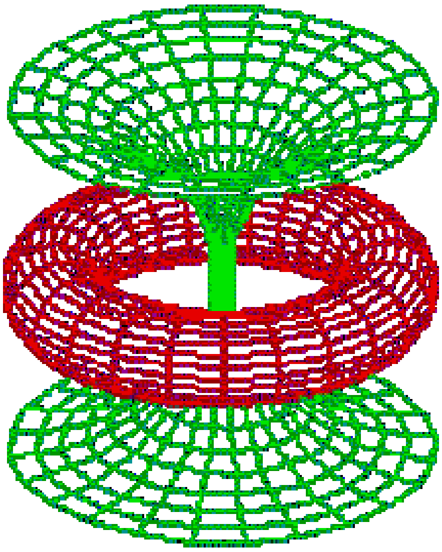

FIGURE 6: Spacetime Structure of a Kerr Hole [20]

If we enter the inner horizon, we can see the spacetime take the shape of the green 2D surface. It looks like an entry and exit, but if nothing can come out of the black hole then how come there is an exit point? If we continue to travel down this path then we can actually come out from the outer event horizon, but it would mathematically be a different space, to be more precise and alternate universe. If we decide to dive back in the Kerr hole, we would eventually come out from the end we previously entered to our own universe but in the past of our universe. Kerr black isn't like a wormhole as it won't collapse without negative mass energy density to keep it open. [20,23,24,27]

Now let's say we enter the Kerr hole again but off from its rotational axis, we would now find ourselves in a ring singularity, the 2D geometry in red. This behaves like a genuine singularity like the Schwarzschild black hole singularity. Mathematically we can circulate around it and keep going in and out of it. [20]

B1. No Hair Theorem

No hair theorem states that all black hole solutions of the Einstein–Maxwell equations of gravitation and electromagnetism in general relativity produces essentially a Schwarzschild black hole and can be completely characterized by only three externally observable classical parameters: mass, electric charge, and angular momentum [17]. Physicist John Archibald Wheeler expressed this idea with the phrase "black holes have no hair",[17] which was the origin of the name. The 'hair' is a metaphor for all other complicated information describing the matter falling into the black hole, which disappears behind the event horizon and thus, becomes permanently inaccessible to external observers. A crucial assumption for the no-hair theorem is that the black hole is isolated; i.e., the spacetime is asymptotically flat and contains no other sources, such as a neighboring accretion disk or plasma matter [28].

However, this assumption doesn't hold in many practical astrophysical situations, such as for a black hole in binary system. The additional sources would contribute to the multipole moments of the spacetime [28]. In their presence, the black hole becomes distorted and the inner geometry of the event horizon varies. Traditionally in such conditions, the no hair theorem cannot hold anymore. Even so, recent developments allow such exceptions. It has been shown that additional parameters have little to no effect on total monopole moment [28]. Although the total multipole moments of the spacetime measured at infinity change, this is solely due to the external sources and not to a different contribution of the black holes themselves. The change in the geometry of the horizon is not reflected in the asymptotic multipole moments [28]. Thus, black holes wearing a wig would still appear bald.

C. Kerr – Newman Blackholes

Kerr – Newman Metric is a solution of Einstein – Maxwell field equations and describes a stationary rotating – charged black hole in asymptotically flat space time.

The Kerr – Newman geometry has both time translation symmetry and rotational symmetry about its 'azimuthal axis (with reference to Boyer – Lindquist Coordinates). A Kerr – Newman Black hole has 2 horizons, inner and outer one and they rotate. Space falls faster than light between the horizons.

Just outside and inside the horizon, some finite regions are present where the world line of an object at rest is space like. This region can be termed as ergosphere. Here nothing can remain at rest. Even the ergosphere has outer and inner region. Objects can escape from within the outer ergosphere (but not from within the outer horizon) but they cannot remain at rest there.

An observer at a distance will see that any object within the outer ergosphere will be dragged around due to the rotation of the black hole. The dragging will be in same direction of rotation of black hole in both outer and inner ergosphere. The outer and inner ergosphere touch the outer and inner horizons at the poles.

Kerr – Newman Black hole possess a charged – ring singularity. The ring singularity is at the focus of the co focal ellipsoids of the Boyer – Lindquist metric. Physically, the singularity is kept open by the centrifugal force. [29, 30]

D. Reissner-Nordström Blackhole

Reissner-Nordström black hole is described by Reissner-Nordstrom metric which is a spherically symmetric static solution of Einstein-Maxwell field equation. It describes a non-rotating black hole having mass and charge in asymptotically flat space time.

The tension produced by radial electric field of the charge creates a gravitational repulsion which is significant at smaller radii. The gravitational repulsion is similar to the centrifugal repulsion inside a rotating black hole.

Reissner-Nordström geometry has 2 horizons – inner and outer horizon. The time coordinate is time like outside the outer horizon; it is space like between the horizons and is again time like inside the inner horizon. The radial coordinate has just the opposite behavior of time coordinate, in these regions. The gravitational repulsion significantly affects the inflow of space.

Gullstrand-Painlevè metric is a part of Reissner-Nordström geometry which gives an idea about space falling into a black hole. Outside the outer horizon, space falls at a speed less than that of light. At the horizon, it falls at speed of light and inside the horizon it falls at speed faster than light. At inner horizon, the flow is reduced back in to speed of light. Inside the inner horizon flow of space slows down and even becomes zero at turnaround radius. Then outflow steps start here. The outflow too takes place at different speeds at different positions of inner and outer horizon. It provides some insights on white holes.

The Reissner-Nordström black hole possess a time like singularity. It is infinitely gravitationally repulsive. It possesses mass as well as charge similar to Kerr – Newman singularity but is quite different from it in other aspects like geometry. [31, 32]

CONCLUSION

Hence to conclude with the Stellar evolution and gravitational collapse we studied the birth of star, what triggers the birth of a star, it's evolution through different stages that is pre-main sequence which is the stage before a stable star is formed. Once a stable star is formed it enters the main sequence. In the main sequence the star is in hydrostatic equilibrium. Once the fuel is exhausted the equilibrium can no longer continue hence the star moves towards the post main sequence phase. In this phase the star collapses to form either a white dwarf star, a neutron star or a singularity based on its mass. Then we looked into the formation of singularity, the three different stages of the formation. We also studied Oppenheimer-Snyder-Datt model and Cosmic Censorship Conjecture. Further added the naked singularities and Hawking radiation. After formation we studied the structure of a singularity which includes different types of black holes and their characteristics explained in detail. Our study ends with the No hair theorem and the classification of black holes on its basis.

ACKNOWLEDGEMENT

We would like to thank our mentor Mr. Samarth Patel for his guidance and support throughout. Along with that we would like to thank the Indian Space Society for giving us the opportunity to work on such an enriching topic. This topic has helped all of us understanding exactly how the field of astronomy.

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