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Elastic Strip Analysis of The Biharmonic Equation for Moments and Deflections of Simply Supported Rectangular Plates Developed from Finite Series Expression for A Suggested Valid Displacement Function

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ABSTRACT
The infinite series method has been used to solve plate problems and the results justified by applying same to beam problems with striking results. The opinion of the author that successful application of the infinite series method to beams when improved upon can be extended to plate problems after considering a plate to comprise four(4) beam strips as indicated by the plate biharmonic equation. The trigonometric function was expressed as the fundamental i.e for the first harmonic alone. Load sharing among the strips was done by the consideration that every strip carried a fraction of the total load of the area obtained by its length multiplied by the length of the perpendicular reaching the plate boundary and such that the sum of all these fractions will equal the total load on the plate. This is quite phenomenal. The application of this shared load to the beams with its substitution into the biharmonic expressions for strips of plates, produced striking results compared to classical methods. A table design tool for plates both with corners held down and corners allowed to lift is evolving.

Keywords: plates; elastic; strip; load-sharing; moments; deflections

INTRODUCTION
The governing equation for rectangular plate has the form.

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q_0}{D} \quad \text{..........................(1)}$$

However, the solution of the above equation has been achieved using the infinite trigonometric series only for a limited class of problems [1]. A major drawback of the series solutions is the yet to be found trigonometric functions to satisfy some load and displacement functions and boundary conditions. This has brought about the introduction of several approximate methods including the finite element methods, the finite strip method, the difference-based finite element method, the grillage analysis and finite difference methods. The yield line theory and the strip method [2, 3, 7, 11 and 12] which are plastic methods have been developed and applied predominantly for the analysis of reinforced concrete slabs. The strip moment ratio theory SMR [4, 6 and 10] to rectangular plates for corners held down and allowed to lift was developed by the author and colleague. The application of the SMR method can be seen in [5, 8 and 9]. This is an elastic strip method. In the SMR method the characterization of the diagonal strip for twisting moment was a major drawback particularly for plates with corners held down. This gave deflections that do not compare to classical results except the poison ratio is kept at zero (0).

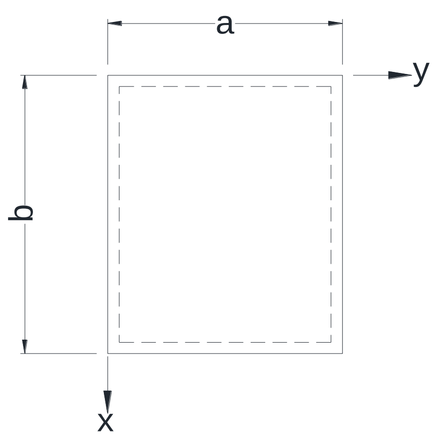


FIGURE 1: Location of coordinate System for Simply Supported Rectangular Plate

This work is an elastic strip method that carefully avoids moment ratio because of the challenges in the SMR method, by separating the individual strip equations from the plate biharmonic equation and making the amplitudes Ax, Ay, Axy and Ayx as equal to each other for the compatibility criterion. The second criterion is that each strip carries the load from the strip length multiplied by the perpendicular to the strip reaching the plate's boundaries. This was only unique for the perpendicular strips. With these two conditions, the load sharing was done and results obtained are striking.

THEORETICAL FRAMEWORK

The equation 1 above has the following definition

q = load intensity
D = Flexural rigidity of plate

$$D = \frac{Eh^3}{12(1-\upsilon^2)} \quad Dx=Dy \qquad (2)$$

$$Dxy = Dyx \frac{Eh^3}{24(1+\upsilon)} \qquad (3)$$

E = Young's Modulus of Elasticity
h = Plate thickness
v = Poisson ratio

Equation (1) is broken down into Harmonic equations given below

$$\frac{\partial^4 w}{\partial x^4} = \frac{-q_x}{D_x} \qquad (4)$$

$$\frac{\partial^4 w}{\partial y^4} = \frac{-q_y}{D_y} \qquad (5)$$

$$\frac{\partial^4 w}{\partial x^2 \partial y^2} = \frac{-q_{xy}}{D_{xy}} \qquad (6)$$

Valid displacement Function for the uniformly distributed load is given below as:

$$w = Amn \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \qquad (7)$$

Where
m = number of half waves in the x direction and
n = number of half waves in the y direction

The infinite series method has been used to solve equations 4, 5, 6 in beam problems with striking results.

It is the opinion of the authors that successful application of the infinite series method to beams as mentioned above can be improved upon and extended to plate problems.

Using the biharmonic operator to find derivative of equation 7;

First, derivative wrt x

$$\nabla^4{}_x = Amn\frac{m^4\pi^4}{a^4}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \qquad (8)$$

By similar differentiation process:

$$\nabla^4{}_y = Amn\frac{n^4\pi^4}{b^4}\sin\frac{m\pi x}{a}.\sin\frac{n\pi y}{b} \qquad (9)$$

$$\nabla^4{}_{yx} = Amn.\frac{m^2\pi^2}{a^2}\frac{n^2\pi^2}{b^2}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \qquad (10)$$

Substituting equations (8), (9) and (10) into equation (1)

Collecting like terms together:

$$\nabla_w{}^4 = Amn\left[\frac{m^4\pi^4}{a^4} + 2\left(\frac{m^2\pi^2}{a^2}.\frac{n^2\pi^2}{b^2}\right) + \frac{n^4\pi^4}{b^4}\right]\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} = \frac{q_0}{D}$$

(11)

In this work, we shall carefully avoid equation 11 and concentrate on equations 8, 9 and 10 which are the expressed form of equations 4,5 and 6. This way we would have turned the plate problem to a beam problem. One in the short span x, the second in the long span y, the third and fourth in the diagonal strip xy and yx.

But first let us simplify the equations from infinite to finite series by making m and n to be unity.

$$\nabla^4{}_x = Ax\frac{\pi^4}{a^4}\sin\frac{\pi x}{a}\sin\frac{\pi y}{b} = \frac{-q_x}{D_x} \qquad (12)$$

By similar differentiation process:

$$\nabla^4{}_y = A_y\frac{\pi^4}{b^4}\sin\frac{\pi x}{a}.\sin\frac{\pi y}{b} = \frac{-q_y}{D_y} \qquad (13)$$

$$\nabla^4{}_{yx} = A_{xy}.\frac{\pi^2}{a^2}\frac{\pi^2}{b^2}\sin\frac{\pi x}{a}\sin\frac{\pi y}{b} = \frac{-q_{xy}}{D_{xy}} \qquad (14)$$

Solving for the amplitude, (A_x, A_y, A_{xy}) involves multiplying both sides of equation (12, 13 and 14) by:

$\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$ and integrating twice.

$$Ax\frac{ab}{4}\left(\frac{\pi}{a}\right)^4 = \frac{-4q_x ab}{D_x\pi^2} \qquad (15)$$

From where

$$A_x = \frac{-16q_x a^4}{D_x\pi^6} \qquad (16)$$

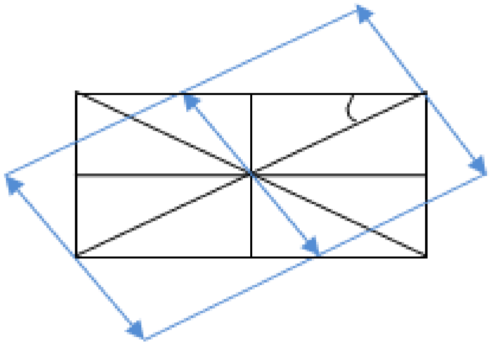
Similarly,

$$A_y = \frac{-16q_y b^4}{D_y\pi^6} \qquad (17)$$

$$A_{xy} = \frac{-16q_{xy}a^2b^2}{D_{xy}\pi^6} \qquad (18)$$

- **Load Sharing**

$qab=q_xab+q_yab+2q_{xy}L_{xy}L_{xyp}$ (19)
where
$L_{xy}= (a^2+b^2)^{0.5}$ (20)
and
L_{xyp} is the perpendicular to L_{xy} as shown below



From similar triangles the length L_{xyp} was obtained as

$$L_{xyp} = \frac{aL_{xy}}{b} \quad (21)$$

An important condition which must be satisfied is that L_{xp} must be chosen such that the aspect ratio of the inclined rectangle for each of the two diagonal strips must be equal to that of the actual plate.
Equation (19) becomes.

$$q = q_x + q_y + 2q_{xy}\frac{L_{xy}^2}{b^2} \quad (22)$$

- **Compatibility**

The compatibility criterion in this work shall be satisfied for the condition that the amplitudes A_x, A_y and A_{xy} must be same and equal

That is
$A_x = A_y = A_{xy}$

Therefore for
$A_x = A_y$

$$A_x = \frac{-16q_x a^4}{D_x \pi^6} = A_y = \frac{-16q_y b^4}{D_y \pi^6} =$$

$$A_{xy} = \frac{-16q_{xy} a^2 b^2}{D_{xy}\pi^6}$$

From where

$$q_x = \frac{n^4 q_y D_x}{D_y} \quad (23)$$

$$q_x = \frac{n^2 q_{xy} D_x}{D_{xy}} \quad (24)$$

$$q_y = \frac{q_x D_y}{n^4 D_x} \quad (25)$$

$$q_{xy} = \frac{q_x D_{xy}}{n^2 D_x} \quad (26)$$

If $n = L_y/L_x$, $n_{xy} = L_{xy}/L_x$

Substituting equations 23, 24,25 and 26 in equation 22 as applicable,

We shall have

$$\frac{q}{q_x} = 1 + \frac{D_y}{n^4 D_x} + 2\frac{n_{xy}^2 D_{xy}}{n^4 D_x} \quad (27)$$

$$\frac{q}{q_y} = 1 + \frac{n^4 D_x}{D_y} + 2\frac{n_{xy}^2 D_{xy}}{D_y} \quad (28)$$

The reciprocal of equations 27 and 28 are the fraction of the total loads carried by the shorter span a and longer span b respectively

$$\alpha = f_x = \frac{q_x}{q} \quad (29)$$

and

$$\beta = f_y = \frac{q_y}{q} \quad (30)$$

$$\gamma = f_{xy} = \frac{q_{xy}}{q} \quad (31)$$

the equation 22 becomes

$$1 = f_x + f_y + 2f_{xy}\frac{L^2_{xy}}{b^2} \quad (32)$$

From where

$$f_{xy} = 0.5[1 - f_x - f_y]\frac{b^2}{L^2_{xy}} \quad (33)$$

DEFLECTIONS

The deflections of Plates are determined by multiplying the primitive beam deflection Δ of the x strip by the x-x strip load factor f_x

The plate deflection Δ_p is

$$\Delta_p = f_x \Delta \quad (34)$$

Where, Δ is the primitive beam deflection of the x-x strip.

BENDING MOMENTS

The bending moments in the plate is given as

$$M_x = D_x\frac{\partial^4 w}{\partial x^4} + \upsilon D_y\frac{\partial^4 w}{\partial y^4} \quad (35)$$

$$M_y = \upsilon D_x\frac{\partial^4 w}{\partial x^4} + D_y\frac{\partial^4 w}{\partial y^4} \quad (36)$$

Where

$$Dx = Dy = \frac{Eh^3}{12(1-\upsilon^2)} \quad (37)$$

$$Dxy = Dyx\frac{Eh^3}{24(1+\upsilon)} \quad (38)$$

The plate moments can now be written from equations 35 and 36 as

$$M_x = f_x M_x + \upsilon f_y M_y \quad (39)$$

and

$$M_x = \upsilon f_x M_x + f_y M_y \quad (40)$$

APPLICATION

The following solution algorithm is convenient for use in a typical problem.

Step 1: Compute plate parameters a, b, L_{xy}, D_x, D_y, D_{xy}
Step 3: Compute quantities f_x, f_y and f_{xy}
Step 5: Compute plate deflection
Step 6: Compute plate moments

• Problems of interest

Determine the moments and maximum deflection of a simply supported rectangular Plate with a uniformly distributed load q for various aspect ratios if corners are

I. held down and
II. allowed to lift.

Also find the effect of varying Poisson ratio ν. Compare results with codes and classical results

RESULTS

The solutions are given below

TABLE 1: showing load fractions for each strip. α, β and γ for the shorter, longer and diagonal strips respectively.

b/a	$2n/n_s$ as length perpendicular to the diagonal length n_s			n_s/n as length perpendicular to the diagonal length n_s		
	α	β	γ	α	β	γ
1	0.2941	0.2941	0.2059	0.2941	0.2941	0.2059
1.1	0.3521	0.2405	0.2037	0.3650	0.2493	0.1928
1.2	0.4074	0.1965	0.1981	0.4337	0.2091	0.1786
1.3	0.4590	0.1607	0.1901	0.4977	0.1742	0.1641
1.4	0.5064	0.1318	0.1809	0.5557	0.1446	0.1498
1.5	0.5495	0.1085	0.1710	0.6072	0.1199	0.1364
1.6	0.5884	0.0898	0.1609	0.6524	0.0995	0.1240
1.7	0.6234	0.0746	0.1510	0.6917	0.0828	0.1128
1.8	0.6547	0.0624	0.1415	0.7257	0.0691	0.1026
1.9	0.6828	0.0524	0.1324	0.7551	0.0579	0.0935
2	0.7080	0.0442	0.1239	0.7805	0.0488	0.0854
3	0.8562	0.0106	0.0666	0.9101	0.0112	0.0393
4	0.9162	0.0036	0.0401	0.9520	0.0037	0.0221
5	0.9455	0.0015	0.0265	0.9702	0.0016	0.0141
10	0.9861	0.0001	0.0069	0.9930	0.0001	0.0035

ns /n is the criterion where the diagonal strips must have an aspect ratio (Lx/Ly=n) of the plate. 2n/ns is the criterion where the diagonal strips must have an aspect ratio such that the perpendicular passes through the corners of the plate.

TABLE 2: showing for modified ns /n results of deflection Δ, Moments Mx in the shorter strip and Moments My in the longer strip for the proposed finite strip method and other known methods in literature.

b/a	Classical Δ	Finite series strip Δ	Classical Mx	Finite series strip Mx	Macus Mx	BS8110	Classical My	Finite series strip My	Macus My	BS 8110 My
1	0.00406	0.0038	0.0479	0.0478	0.0362	0.055	0.0479	0.0478	0.0362	0.056
1.2	0.00564	0.0053	0.0627	0.0615	0.0512	0.074	0.0501	0.0511	0.036	0.056
1.4	0.00705	0.0066	0.0755	0.0730	0.0656	0.087	0.0502	0.0522	0.0338	0.056
1.6	0.0083	0.0077	0.0862	0.0822	0.0783	0.0964	0.0492	0.0519	0.0302	0.056
1.8	0.00931	0.0085	0.0948	0.0894	0.0866	0.105	0.0479	0.0511	0.0268	0.056
2	0.0101	0.0092	0.1017	0.0951	0.0995	0.111	0.0464	0.0500	0.0233	0.056
3		0.0111	0.1189	0.1106			0.0406	0.0450		
4		0.0119	0.1235	0.1167			0.0384	0.0422		
5		0.0123	0.1246	0.1196			0.0375	0.0406		
10		0.0128	0.125	0.1236			0.0375	0.0383		

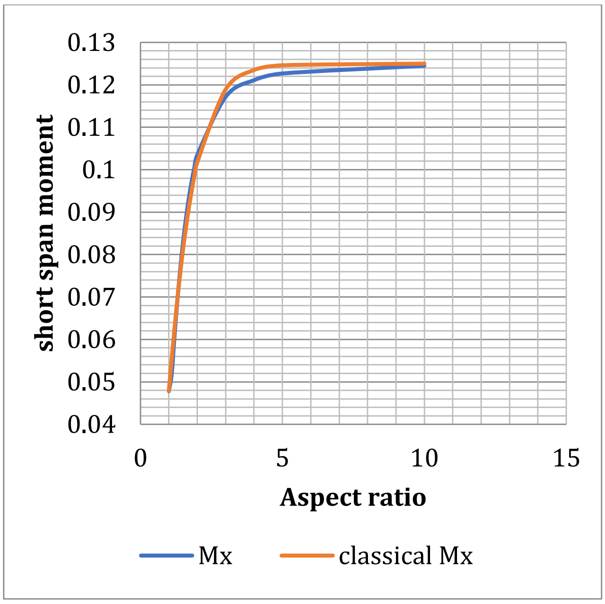


FIGURE 1: showing moments in longer span

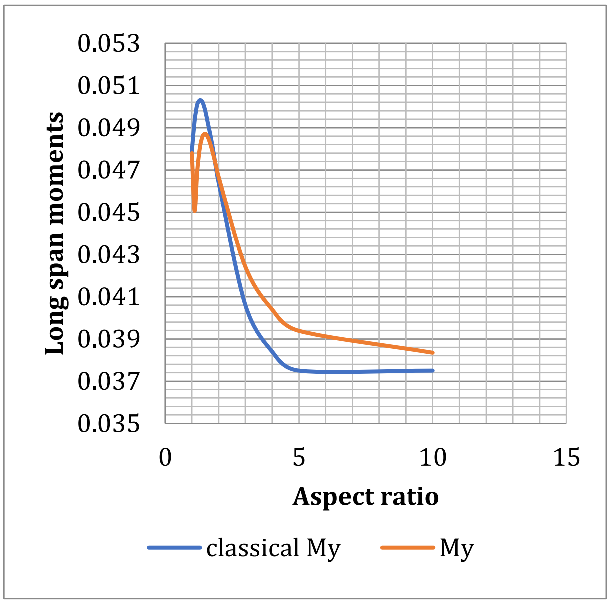


FIGURE 2: showing moments in shorter span

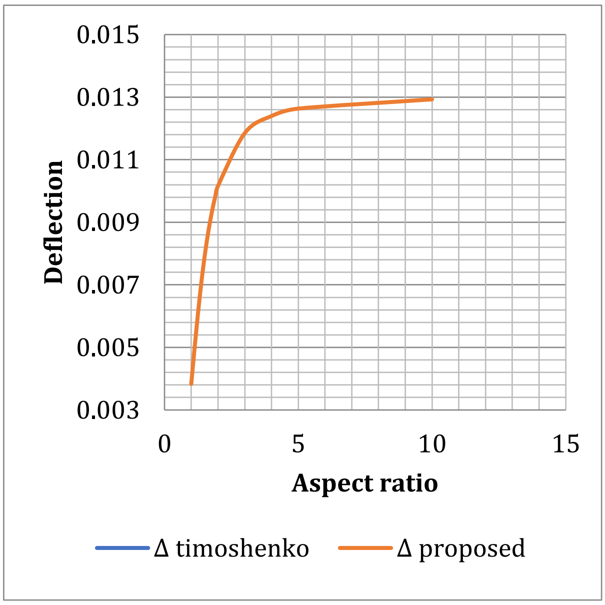


FIGURE 3: showing deflections of plate

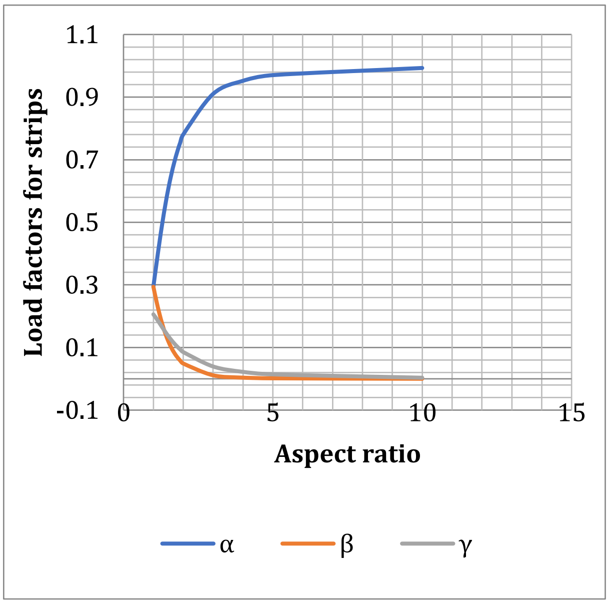


FIGURE 4: showing load fractions in the strips

TABLE 3: showing for modified n_s /n results of deflection Δ, Moments Mx in the shorter strip and Moments My in the longer strip for the proposed finite strip method.

b/a	Classical Δ	Finite series strip Δ	Classical Mx	Finite series strip Mx	My classical	Finite series strip My
1	0.00406	0.0038	0.0479	0.0478	0.0479	0.0478
1.1		0.0048	0.0554	0.0569	0.0493	0.0511
1.2	0.00564	0.0056	0.0627	0.0655	0.0501	0.0534
1.3		0.0065	0.0694	0.0732	0.0503	0.0547
1.4	0.00705	0.0072	0.0755	0.0801	0.0502	0.0553
1.5		0.0079	0.0812	0.0860	0.0498	0.0554
1.6	0.0083	0.0085	0.0862	0.0911	0.0492	0.0551
1.7		0.0090	0.0908	0.0954	0.0486	0.0545
1.8	0.00931	0.0094	0.0948	0.0991	0.0479	0.0538
1.9		0.0098	0.0985	0.1022	0.0471	0.0530
2	0.0101	0.0102	0.1017	0.1049	0.0464	0.0522

TABLE 4: showing for modified 2n/ns results of deflection Δ, Moments Mx in the shorter strip and Moments My in the longer strip for the proposed finite strip method.

b/a	Classical Δ	Finite series strip Δ	Classical Mx	Finite series strip Mx	My classical	Finite series strip My
1	0.00406	0.0038	0.0479	0.0478	0.0479	0.0478
1.2	0.00564	0.0056	0.0627	0.0609	0.0501	0.0469
1.4	0.00705	0.0072	0.0755	0.0765	0.0502	0.0486
1.6	0.0083	0.0085	0.0862	0.0884	0.0492	0.0485
1.8	0.00931	0.0094	0.0948	0.0971	0.0479	0.0477
2	0.0101	0.0102	0.1017	0.1034	0.0464	0.0467
3		0.0119	0.1189	0.1172	0.0406	0.0424
4		0.0124	0.1235	0.1211	0.0384	0.0404
5		0.0126	0.1246	0.1227	0.0375	0.0394
10		0.0129	0.125	0.1245	0.0375	0.0383

- **Plates with corners lifting**

Plates with corners not held down were also handled very easily with the model. Here the diagonal strips are considered not active and the load is shared to only the shorter and longer strips. The table 5 below gives the results and Poisson ratio is taken as zero.

TABLE 5: showing results of deflection Δ, Moments Mx in the shorter strip and Moments My in the longer strip for the proposed finite strip method and other known methods in literature of plates with corners not held down.

b/a	α	β	Δ proposed	Bs8110 Mx	Finite strip Mx	Macus Mx	SMR Mx	Bs8110 My	Finite strip My	Macus My	SMR My
1	0.50	0.50	0.0065	0.062	0.0625	0.0625	0.0625	0.0620	0.0625	0.0625	0.0625
1.2	0.68	0.33	0.0088	0.084	0.0843	0.0844	0.0843	0.0590	0.0586	0.0406	0.0586
1.4	0.79	0.21	0.0103	0.099	0.0992	0.0991	0.0992	0.0510	0.0506	0.0259	0.0506
1.6	0.87	0.13	0.0113	0.1085	0.1085	0.1085	0.108	0.0420	0.0424	0.0165	0.0424
1.8	0.91	0.087	0.0119	0.114	0.1141	0.1144	0.114	0.0352	0.0352	0.0121	0.0352
2	0.94	0.059	0.0123	0.118	0.1176	0.1176	0.1176	0.0290	0.0294	0.0074	0.0294
3	0.99	0.012	0.0129		0.1235	0.1235	0.1234		0.0137	0.0015	0.0137
4	0.996	0.004	0.0130		0.1245		0.1245		0.0078		.0078
5	0.998	0.00	0.0130		0.1248		0.1248		0.0050		
10	1.0	0.00	0.0130		0.1250		0.1250		0.0012		

DISCUSSION OF RESULTS

The results in tables 2, 3 and 4 where analyzed based on simple statistics for plates with varying aspect ratio from 1 to 10 and compared with the classical results by Levy in (i). the percentage difference and simple ratio are given in table 6 below

TABLE 6: showing comparison of the proposed finite series elastic strip method to the classical infinite series method of levy.

Proposed perpendicular length of diagonal strips	% difference with classical Levy for Mx	Ratio of proposed to classical Levy for Mx	% difference with classical Levy for My	Ratio of proposed to classical Levy for My
ns/n	-2.80	1.03	-9.07	1.09
2n/ns	3.92	0.96	-5.27	1.053
Modified ns/n	0.036	1.000667	0.6502	0.993

From the table it can be seen that the proposed perpendicular length of the diagonals of ns/n, which equally gives an aspect ratio n of the plate gives striking results which are improved by the modification

$$M_x = D_x \frac{\partial^4 w}{\partial x^4} + \frac{\upsilon}{n_{sp}} D_y \frac{\partial^4 w}{\partial y^4} \quad (41)$$

Modified from equation 35 and

$$M_y = \frac{\upsilon}{n_{sp}} D_x \frac{\partial^4 w}{\partial x^4} + D_y \frac{\partial^4 w}{\partial y^4} \quad (42)$$

Modified from equation 36

Where

$$n_{sp} = \frac{{n_s}^2}{n^2}$$

The plates with corners held down produced almost same results with codes method. It appears that the classical method never treated these plate types. In both corners held down and corners allowed to lift the deflection results are also striking.

CONCLUSION AND RECOMMENDATIONS

The following conclusion and recommendations are drawn from the work

- **Conclusion**

A table method of plate analysis using finite series method to share the loads on the plate to the four strips in a plate as represented in the biharmonic equation was developed in the paper. The compatibility criterion of equal Amplitude in every strip and the load sharing formula, such that the sum of load carried by each strip equals unity was used in executing this work. These load fractions when multiplied by their respective strip moments and made to satisfy equations 35 and 36 gives the respective bending moments. The deflection of the plate is simply that of the strip multiplied by the load fraction of that strip. The results obtained are striking. Tables and charts have also been presented for easy design of plates. The Strip Moment Ratio (SMR) method also developed by the author basically has characterization of twisting moments has an area that has not still received adequate research attention after about two decades of its development. The proposed finite series method developed in this work has basically two to three advantages of deliberately avoiding the (i) twisting moments which has not been adequately characterized in the SMR method and (ii) time and manipulations in the infinite series method which in itself requires space and time among other things.

- **Recommendation**

A method to clearly and practically solve plate problems without heavy mathematical manipulations which has been developed here can be extended to other plates with different shapes, support systems and loading and it is recommended.

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