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Distribution and Randomness of the Numbers in $\sqrt{2}$

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ABSTRACT

This study investigated the distribution and randomness of numbers in square root 2. Rstudio was used in extracting the square root of 2 up to 1,000 digits. The Chi-square test is used for testing the uniformity of the numbers, and the runs test is used for testing the randomness of the numbers. Results show that the distributions of the generated numbers for n = 100 and for n = 1,000 are uniform. The sequence of numbers generated for n = 100 and n = 1,000 appears to be random.

Keywords: distribution; randomness; irrational numbers; chi-square goodness of fit test; runs test.

INTRODUCTION

In mathematics, a square root is a factor of a number that gives the original number when multiplied by itself. Hence, the concept of a radical expression, or more precisely square and cube roots, is said to have originated in Egypt and Babylonia around 1800 B.C. In addition, the concept of square and cube roots appears to have originated in geometric representations, notably in the discovery of the length of a square or cube's side with a known area [5].

In the same manner, the ancient Babylonians are also credited with being the first civilization to record exact approximations of non-square numbers, such as square root of 2, which would later be referred to as irrational numbers [3]. In addition, YBC 7289 tablet is one of the most well-known Old Testament figures from the Yale Babylonian Collection. The actual provenance and age of this Babylonian mathematical clay tablet are uncertain; however, the circular form of the tablet and the paleography implies that it was made in Babylon, somewhere in southern Mesopotamia, written by a trainee writer, a modern Iraq sometime in the first third of the second millennium BC [4].

In addition, irrational numbers are any real number that cannot be expressed as the quotient of two integers. It can be expressed as an infinite decimal expansion with no repeating digits and the square root of 2 is the best example [1, 7].

The distribution of numbers in radicals is useful in real life. A rational exponent is an exponent in the form of a fraction. Any expression that contains the square root of a number is a radical expression. Both have real world applications in fields like architecture, carpentry and masonry. Radical expressions are utilized in financial industries to calculate formulas for depreciation, home inflation and interest. Electrical engineers also use radical expressions for measurements and calculations. Biologists compare animal surface areas with radical exponents for size comparisons in scientific research [2].

In statistics, distribution is a function that provides the chance that the variables observed value will fall within any given range of potential values. It also tells how often each value occurs and normal distribution is the most widely used distribution [6, 8]. The sequence of Random Numbers plays an important role in cryptology particularly in coding and decoding information.

This study aims to determine the distribution and randomness of numbers in the square root of 2 and to verify if the sequence of numbers in the square root of 2 is uniform and random. The scope of this study limits its coverage to n = 100 and n = 1000 generated numbers in the square root of 2. The generated numbers were used for testing for uniformity and randomness.

RESULTS AND DISCUSSION

Tables 1 and 2 give a brief summary of the characteristics of the numbers generated, and figures 1 and 2 show the observed and expected value of the numbers generated in the square root of 2.

TABLE 1: Distribution of Numbers in $\sqrt{2}$ (n = 100)

n = 100	
NUMBERS	FREQUENCY
1	8
2	8
3	11
4	9
5	7
6	10
7	17
8	12
9	8
0	10
TEST FOR UNIFORMITY	χ^2 =7.6; *p*-value = 0.574903
TEST FOR RANDOMNESS	*z-value = -0.7205; p-value = 0.47121*
Mean (expected) = 4.5	Mean (observed) = 4.75
Variance (expected) = 8.25	Variance(observed)=8.11

Table 1 exhibits the frequency distribution of numbers extracted in square root of 2. A total of 100 digits are extracted. The frequencies of numbers in the extracted square root of 2 for n = 100 ranged from 7 to 17. The computed value χ^2 = 7.6 with a p-value of 0.574903 justifies the uniformity. The z-value of -0.7205 with a p-value of 0.47121 braces the sequence is random. The generated numbers show an observed mean of 4.75 and a variance of 8.11, which are very close to the expected values.

FIGURE 1: Clustered Column Chart for the Observed and Expected Frequencies (n = 100)

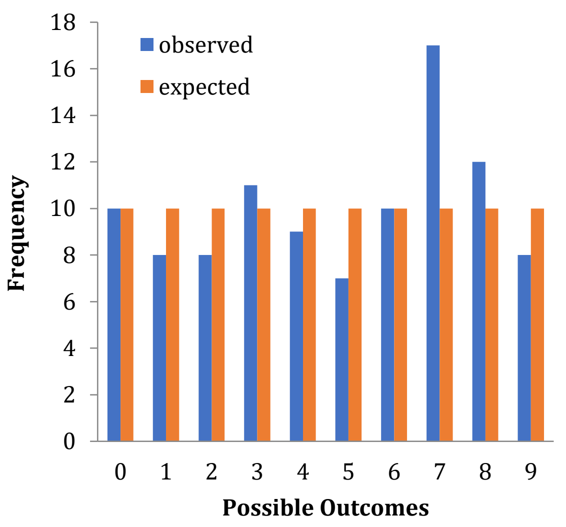

Figure 1 is the frequency bar graph of n = 100. The highest appearance in 100 extracted values of square root of 2 is 7 with a frequency of 17 and the smallest is 5 with a frequency of 7. The observed and expected frequencies are almost the same.

TABLE 2: Distribution of Numbers in $\sqrt{2}$ (n = 1000)

n = 1000	
NUMBERS	FREQUENCY
1	99
2	108
3	82
4	100
5	104
6	90
7	104
8	113
9	92
0	108
TEST FOR UNIFORMITY	χ^2 = 8.18; p-value = 0.518106
TESTFOR RANDOMNESS	*z-value = 1.22474; p-value = 0.22067*
Mean (expected) = 4.5	Mean (observed) = 4.481
Variance (expected)= 8.25	Variance (observed)=8.41

Table 2 presents the frequency distribution of numbers extracted in square root of 2 with total of 1000 digits extracted. Frequencies of numbers in the extracted square root of 2 for n = 1000 ranged from 82 to 113. The computed value χ^2 = 8.18 with a p-value of 0.518106 suffices that it is uniformly distributed. In the test for randomness, the z-value is 1.22474 with a p-value of 0.22067 supports the sequence is random. The generated numbers show an observed mean of 4.481 and a variance of 8.41 have a very minimal difference to the expected values.

FIGURE 2: Clustered Column Chart for the Observed and Expected Frequencies *(n* = 1000)

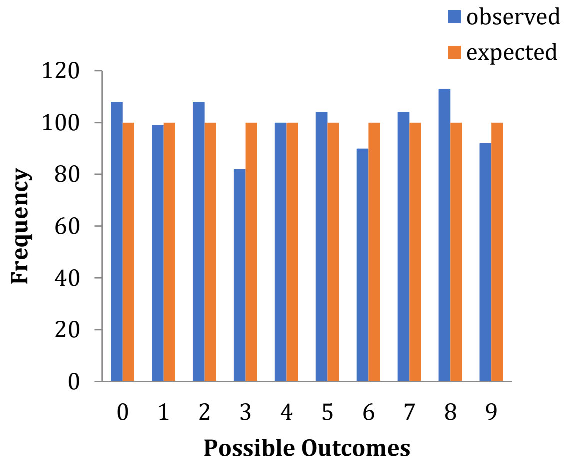

Figure 2 is the frequency bar graph of n = 1000. The highest appearance in 1000 extracted values of square root of 2 is 8 with a frequency of 113 and the smallest is 3 with a frequency of 82. Observed frequencies are closed to the expected frequencies.

CONCLUDING REMARKS

By the chi-square goodness of fit test, the distribution of numbers in the square root of 2 for n = 100 and n = 1000 is uniform. With the runs test, the sequence for n = 100 and n = 1000 is also random. We recommend further study on generating numbers and testing the distribution for uniformity and randomness on other irrational numbers.

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APPENDIX

(Extracted value of square root of 2 in *n* = 100 and *n* = 1000)

n = 100
1.4142135623730950488016887242096980785696718753769480731766797379907324784621070388503875343276415727
n = 1000
1.414213562373095048801688724209698078569671875376948073176679737990732478462107038850387534 32764157273501384623091229702492483605585073721264412149709993583141322266592750559275579995 05011527820605714701095599716059702745345968620147285174186408891986095523292304843087143214 50839762603627995251407989687253396546331808829640620615258352395054745750287759961729835575 22033753185701135437460340849884716038689997069900481503054402779031645424782306849293691862 15805784631115966687130130156185689872372352885092648612494977154218334204285686060146824720 77143585487415565706967765372022648544701585880162075847492265722600208558446652145839889394 43709265918003113882464681570826301005948587040031864803421948972782906410450726368813137398 55256117322040245091227700226941127573627280495738108967504018369868368450725799364729060762 99694138047565482372899718032680247442062926912485905218100445984215059112024944134172853147 81058036033710773091828693147101711116839165817268894197587165821521282295184884720