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# Pseudo Random Numbers Generated By Using Mixture Of Modular Function And Tent Map

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**ABSTRACT**

In this paper, we develop an algorithm in generating random numbers from the mixture of the modular function and the tent map. Results show that there are more random numbers generated from the mixture compared to the random numbers generated purely from the modular function and the tent map. Moreover, majority of the distributions of the generated random numbers for the mod100 are uniform. However, the distributions of the generated random numbers for the mod1000 are not uniform. Lastly, the sequences of random numbers generated for divisor of mod100 and mod1000 behave to be random.

***Keywords:*** random numbers; modular function; tent map; algorithm

## INTRODUCTION

The use of random numbers in statistics has extended more than in random sampling and random assignment of treatments to experimental units [1]. They are now being used in simulation studies such as in stochastic processes, engineering, biological sciences and cryptography. In cryptography, the requirements for "randomness" are more stringent than for ordinary applications in simulation [1].

Things that make random numbers are generically called Random Number Generators (RNGs), and they are classified into two major types: Pseudo-Random Numbers Generators (PRNGs) and True Random Number Generators (TRNGs). PRNGs are deterministic algorithms, while TRNGs are nondeterministic systems [2].

The digital computer cannot generate true random numbers, and it is not convenient to connect the computer to some external source of random events [1]. However, this can be overcome if there are some sources of pseudorandom numbers in which samples can be drawn from some known distributions.

True random numbers have some disadvantages, among them are: 1. The random numbers are to be extracted from natural resources which demand a lot of hardware and are expensive to set up. 2. The generation speed of random numbers may not be sufficient for the application. 3. Entropy sources can be influenced by the changes in the physical environment of the device like changes in the temperature, changes in the voltage or frequency of the power supply, exposure to radiation, etc.. 4. An intruder can control some of the parameters that may affect the entropy value [4].

It can be noted that one of the most important methodologies in operations research, numerical analysis and statistics is the deployment of random experiments for solving problems [3]. The method of random experiments has undergone since the end of 1940's, a fast development for in those years the rapid growth in the computational capacities of the computers made the executions of the random experiments in really large numbers possible [3].

As of this time, there are many pseudo random numbers generators being developed: Some are Middle Square Method, Linear Feedback Shift Registers and Xorxhiff Generators [4]. The extraction of the numbers in the square root of 2 is also a potential can be a good method of generating random numbers [5].

Recently, Padua [6] hinted that the tent map can generate U(0.1) variates and the logistic map can be a good substitute for the arcsine law.

Modular arithmetic can be handled mathematically be introducing a congruence relation on the integers that is compatible with the operations of the ring of integers: addition, subtraction, and multiplication. For a fixed modulus n, it is defined as follows. Two integers a and b are said to be congruent modulo n, if their difference (a-b) is an integer multiple of n. If this is the case, it is expressed as: $a \equiv b \pmod{n}$. [7]

On the other hand, the tent map, formed by the iteration of the output value in a transformation of modular function, can be used for the generation of random numbers, given by the equation

$$T(x) = \begin{cases} 2x, & for\ 0 \le x < \frac{1}{2} \\ 2 - 2x, & for\ \frac{1}{2} \le x < 1 \end{cases}$$

In this paper, we present an algorithm in generating random pseudo numbers by using the mixture of modular function and tent map that can automatically create long runs of numbers.

## THE ALGORITHMS

This paper ventures to generate random numbers by getting the mixture of the modular and tent map. Important developments, results and findings of the study are the following:

**(1) Algorithm for Modular Function.** These are the following steps in generating random numbers by using the Modular:

1. Input $X_n, n = 0$.
2. Compute $X_{n+1} = \big(X_n \cdot (P_1 + P_2)\big) mod N$

**(2) Algorithm for Tent Map.** These are the following steps in generating random numbers by using the Tent Map:

1. Input $X_n, n = 0$.
2. Let $X_n = Y_n$.
3. Convert $Y_n$ to a number from 0 to 1 by dividing it by N, where N is the number of digits from the decimal point of $X_n$.
4. Compute $Y_{n+1} = \begin{cases} 2Y_n, & for\ 0 \le Y_n < \frac{1}{2} \\ 2 - 2Y_n, & for\ \frac{1}{2} \le Y_n < 1 \end{cases}$
5. Convert $Y_{n+1}$ to a whole number by multiplying it by $N$.

**(3) Algorithm for Mixtures of Modular and Tent Map.** These are the following steps in generating random numbers by using the mixtures of Modular and Tent map:

1. Input $X_n, n = 0$.
2. Compute $X_{n+1} = (X_n \bullet P_1 + P_2)\ mod\ N$.
3. Let $X_n = Y_n, n = 0$.
4. Convert $Y_n$ to number from 0 to 1 by dividing it by $N$. That is, $W_n = \dfrac{Y_n}{N}$.
5. Compute $W_{n+1} = \begin{cases} 2W_n, & for 0 \le W_n < \frac{1}{2} \\ 2 - 2W_n, & for \frac{1}{2} \le W_n < 1 \end{cases}$.
6. Convert $W_{n+1}$ to a whole number by multiplying it by $N$. That is, $Z_{n+1} = NW_{n+1}$
7. Compute $\frac{X_{n+1} + Z_{n+1}}{2}$. This is the random number generated.
8. Go to step 2 for the computation of $X_{n+1}$. Go to step 4 for the computation of $Z_{n+1}$.

We consider the following as examples:

$P_1 = 263, P_2 = 71, X_0 = 79.49,33,85.66\ for\ N = 100$ and $X_0 = 799.499,330,850.660\ for\ N = 1000$.

**THE RESULTS**

Tables 1 and 2 give a brief summary of the characteristics of the random numbers generated.

**TABLE 1:** Characteristics of the Random Numbers Generated N=100.

| Seeds $X_0$ | | 33 | 49 | 66 | 79 | 85 |
|---|---|---|---|---|---|---|
| Number of Random Numbers Generated | Modular | 20 | 20 | 20 | 20 | 20 |
| | Tent Map | 12 | 12 | 11 | 12 | 12 |
| | Mixtures of Modular and Tent Map | 22 | 22 | 21 | 22 | 22 |
| Test for Uniformity | | $\chi^2 = 2.5^*$ | $\chi^2 = 12.5$ | $\chi^2 = 4^*$ | $\chi^2 = 5^*$ | $\chi^2 = 4^*$ |
| Test for Randomness | | $z = 0.42^{**}$ | $z = -1.35^{**}$ | $z = -0.42^{**}$ | $z = 0.46^{**}$ | $z = 0.47^{**}$ |

*-Distribution is uniform
**- Sequence of numbers vary randomly

Table 1 shows the characteristics of the random numbers generated by Modular, Tent Map, and the Mixture of the Modular and Tent Map where *N*=100. It can be observed that there are more random numbers generated by the mixture of the Modular and Tent Map compared to the Modular and Tent Map.

$\chi^2$ values show that majority of the distributions of the random numbers generated are uniform. *Z*-values also show that the sequence of numbers generated varies randomly at 0.05 probability level.

**TABLE 2:** Characteristics of the Random Numbers Generated *N*=1000.

| Seeds $X_0$ | | 330 | 490 | 660 | 790 | 850 |
|---|---|---|---|---|---|---|
| Number of Random Numbers Generated | Modular | 100 | 100 | 100 | 100 | 100 |
| | Tent Map | 12 | 12 | 11 | 12 | 12 |
| | Mixtures of Modular and Tent Map | 102 | 102 | 101 | 102 | 102 |
| Test for Uniformity | | $\chi^2 = 36.5$ | $\chi^2 = 31.7$ | $\chi^2 34.8$ | $\chi^2 = 27.3$ | $\chi^2 = 36.5$ |
| Test for Randomness | | $z = -0.79^{**}$ | $z = 0.197^{**}$ | $z = 0.8^{**}$ | $z = -0.99^{**}$ | $z = 099^{**}$ |

*-Distribution is uniform
**- Sequence of numbers vary randomly

Table 2 shows the characteristics of the random numbers generated by Modular, Tent Map, and the Mixture of the Modular and Tent Map where $N$=1000. It can be observed that there are more random numbers generated by the mixture of the Modular and Tent Map compared to the Modular and Tent Map. $\chi^2$ values show that the distributions of the random numbers generated are not uniform. $Z$-values also show that the sequence of numbers generated varies randomly at 0.05 probability level.

**CONCLUDING REMARKS**

This study has a very interesting results where we can get more random numbers generated by the Mixture of the Modular and Tent Map compared to the Modular and Tent Map. Majority of the distribution of the generated random numbers for $N$ = 100 is uniform. However, the distribution of the generated random numbers for $N$ = 1000 is not uniform. The sequence of random numbers generated for $N$ = 100 and $N$ = 1000 behaves to be random. Hence, we recommend further study on generating random numbers particularly on the mixture of modular and chaotic functions and an investigation of the distribution of the random numbers generated by the mixture of modular and tent map.

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**APPENDIX A (Random Numbers Generated)**

| Seed value =79 | | | Seed value=49 | | |
|---|---|---|---|---|---|
| **Modular** | **Chaos** | **Mixture** | **Modular** | **Chaos** | **Mixture** |
| 48 | 42 | 45 | 58 | 98 | 78 |
| 95 | 84 | 90 | 25 | 4 | 15 |
| 56 | 32 | 44 | 46 | 8 | 27 |
| 99 | 64 | 82 | 69 | 16 | 43 |
| 8 | 72 | 40 | 18 | 32 | 25 |
| 75 | 56 | 66 | 5 | 64 | 35 |
| 96 | 88 | 92 | 86 | 72 | 79 |
| 19 | 24 | 22 | 89 | 56 | 73 |
| 68 | 48 | 58 | 78 | 88 | 83 |
| 55 | 96 | 76 | 85 | 24 | 55 |
| 36 | 8 | 22 | 26 | 48 | 37 |
| 39 | 16 | 28 | 9 | 96 | 53 |
| 28 | 32 | 30 | 38 | 8 | 23 |
| 35 | 64 | 49 | 65 | 16 | 41 |
| 76 | 72 | 74 | 66 | 32 | 49 |
| 59 | 56 | 57 | 29 | 64 | 47 |
| 88 | 88 | 88 | 98 | 72 | 85 |
| 15 | 24 | 19 | 45 | 56 | 51 |
| 16 | 48 | 32 | 6 | 88 | 47 |
| 79 | 96 | 87 | 49 | 24 | 37 |
| 48 | 8 | 28 | 58 | 48 | 53 |
| 95 | 16 | 56 | 25 | 96 | 61 |
| 56 | 32 | 44 | 46 | 8 | 27 |
| 99 | 64 | 82 | 69 | 16 | 43 |
| 8 | 72 | 40 | 18 | 32 | 25 |

| Seed Value=33 | | | Seed value=85 | | |
|---|---|---|---|---|---|
| **Modular** | **Chaos** | **Mixture** | **Modular** | **Chaos** | **Mixture** |
| 50 | 66 | 58 | 26 | 26 | 26 |
| 21 | 68 | 45 | 9 | 52 | 31 |
| 94 | 64 | 79 | 38 | 96 | 67 |
| 93 | 72 | 83 | 65 | 8 | 37 |
| 30 | 56 | 43 | 66 | 16 | 41 |
| 61 | 88 | 75 | 29 | 32 | 31 |
| 14 | 24 | 19 | 98 | 64 | 81 |
| 53 | 48 | 51 | 45 | 72 | 59 |
| 10 | 96 | 53 | 6 | 56 | 31 |
| 1 | 8 | 5 | 49 | 88 | 69 |
| 34 | 16 | 25 | 58 | 24 | 41 |
| 13 | 32 | 23 | 25 | 48 | 37 |
| 90 | 64 | 77 | 46 | 96 | 71 |
| 41 | 72 | 57 | 69 | 8 | 39 |
| 54 | 56 | 55 | 18 | 16 | 17 |
| 73 | 88 | 81 | 5 | 32 | 19 |
| 70 | 24 | 47 | 86 | 64 | 75 |
| 81 | 48 | 65 | 89 | 72 | 81 |
| 74 | 96 | 85 | 78 | 56 | 67 |
| 33 | 8 | 21 | 85 | 88 | 87 |
| 50 | 16 | 33 | 26 | 24 | 25 |
| 21 | 32 | 27 | 9 | 48 | 29 |
| 94 | 64 | 79 | 38 | 96 | 67 |
| 93 | 72 | 83 | 65 | 8 | 37 |
| 30 | 56 | 43 | 66 | 16 | 41 |

| Seed value=66 | | | Seed value=790 | | |
|---|---|---|---|---|---|
| **Modular** | **Chaos** | **Mixture** | **Modular** | **Chaos** | **Mixture** |
| 29 | 68 | 49 | 841 | 420 | 631 |
| 98 | 64 | 81 | 254 | 840 | 547 |
| 45 | 72 | 59 | 873 | 320 | 597 |
| 6 | 56 | 31 | 670 | 640 | 655 |
| 49 | 88 | 69 | 281 | 720 | 501 |
| 58 | 24 | 41 | 974 | 560 | 767 |
| 25 | 48 | 37 | 233 | 880 | 557 |
| 46 | 96 | 71 | 350 | 240 | 295 |
| 69 | 8 | 39 | 121 | 480 | 301 |
| 18 | 16 | 17 | 894 | 960 | 927 |
| 5 | 32 | 19 | 193 | 80 | 136 |
| 86 | 64 | 75 | 830 | 160 | 495 |
| 89 | 72 | 81 | 361 | 320 | 341 |
| 78 | 56 | 67 | 14 | 640 | 327 |
| 85 | 88 | 87 | 753 | 720 | 737 |
| 26 | 24 | 25 | 110 | 560 | 335 |
| 9 | 48 | 29 | 1 | 880 | 441 |
| 38 | 96 | 67 | 334 | 240 | 287 |
| 65 | 8 | 37 | 913 | 480 | 697 |

| | | | | | |
|---|---|---|---|---|---|
| 66 | 16 | 41 | 190 | 960 | 575 |
| 29 | 32 | 31 | 41 | 80 | 61 |
| 98 | 64 | 81 | 854 | 160 | 507 |
| 45 | 72 | 59 | 673 | 320 | 497 |
| 6 | 56 | 31 | 70 | 640 | 355 |
| | | | 481 | 720 | 601 |
| | | | 574 | 560 | 567 |
| | | | 33 | 880 | 457 |
| | | | 750 | 240 | 495 |
| | | | 321 | 480 | 401 |
| | | | 494 | 960 | 727 |
| | | | 993 | 80 | 537 |
| | | | 230 | 160 | 195 |
| | | | 561 | 320 | 441 |
| | | | 614 | 640 | 627 |
| | | | 553 | 720 | 637 |
| | | | 510 | 560 | 535 |
| | | | 201 | 880 | 541 |
| | | | 934 | 240 | 587 |
| | | | 713 | 480 | 597 |
| | | | 590 | 960 | 775 |
| | | | 241 | 80 | 161 |
| | | | 454 | 160 | 307 |
| | | | 473 | 320 | 397 |
| | | | 470 | 640 | 555 |
| | | | 681 | 720 | 701 |
| | | | 174 | 560 | 367 |
| | | | 833 | 880 | 857 |
| | | | 150 | 240 | 195 |
| | | | 521 | 480 | 501 |
| | | | 94 | 960 | 527 |
| | | | 793 | 80 | 437 |
| | | | 630 | 160 | 395 |
| | | | 761 | 320 | 541 |
| | | | 214 | 640 | 427 |
| | | | 353 | 720 | 537 |
| | | | 910 | 560 | 735 |
| | | | 401 | 880 | 641 |
| | | | 534 | 240 | 387 |
| | | | 513 | 480 | 497 |
| | | | 990 | 960 | 975 |
| | | | 441 | 80 | 261 |
| | | | 54 | 160 | 107 |
| | | | 273 | 320 | 297 |
| | | | 870 | 640 | 755 |
| | | | 881 | 720 | 801 |
| | | | 774 | 560 | 667 |
| | | | 633 | 880 | 757 |
| | | | 550 | 240 | 395 |

| | | | | | |
|---|---|---|---|---|---|
| | | | 721 | 480 | 601 |
| | | | 694 | 960 | 827 |
| | | | 593 | 80 | 337 |
| | | | 30 | 160 | 95 |
| | | | 961 | 320 | 641 |
| | | | 814 | 640 | 727 |
| | | | 153 | 720 | 437 |
| | | | 310 | 560 | 435 |
| | | | 601 | 880 | 741 |
| | | | 134 | 240 | 187 |
| | | | 313 | 480 | 397 |
| | | | 390 | 960 | 675 |
| | | | 641 | 80 | 361 |
| | | | 654 | 160 | 407 |
| | | | 73 | 320 | 197 |
| | | | 270 | 640 | 455 |
| | | | 81 | 720 | 401 |
| | | | 374 | 560 | 467 |
| | | | 433 | 880 | 657 |
| | | | 950 | 240 | 595 |
| | | | 921 | 480 | 701 |
| | | | 294 | 960 | 627 |
| | | | 393 | 80 | 237 |
| | | | 430 | 160 | 295 |
| | | | 161 | 320 | 241 |
| | | | 414 | 640 | 527 |
| | | | 953 | 720 | 837 |
| | | | 710 | 560 | 635 |
| | | | 801 | 880 | 841 |
| | | | 734 | 240 | 487 |
| | | | 113 | 480 | 297 |
| | | | 790 | 960 | 875 |
| | | | 841 | 80 | 461 |
| | | | 254 | 160 | 207 |
| | | | 873 | 320 | 597 |
| | | | 670 | 640 | 655 |
| | | | 281 | 720 | 501 |

| Seed value =490 | | | Seed value=330 | | |
|---|---|---|---|---|---|
| **Modular** | **Chaos** | **Mixture** | **Modular** | **Chaos** | **Mixture** |
| 941 | 980 | 961 | 861 | 660 | 761 |
| 554 | 40 | 297 | 514 | 680 | 597 |
| 773 | 80 | 427 | 253 | 640 | 447 |
| 370 | 160 | 265 | 610 | 720 | 665 |
| 381 | 320 | 351 | 501 | 560 | 531 |
| 274 | 640 | 457 | 834 | 880 | 857 |
| 133 | 720 | 427 | 413 | 240 | 327 |
| 50 | 560 | 305 | 690 | 480 | 585 |

| | | | | | |
|---|---|---|---|---|---|
| 221 | 880 | 551 | 541 | 960 | 751 |
| 194 | 240 | 217 | 354 | 80 | 217 |
| 93 | 480 | 287 | 173 | 160 | 167 |
| 530 | 960 | 745 | 570 | 320 | 445 |
| 461 | 80 | 271 | 981 | 640 | 811 |
| 314 | 160 | 237 | 74 | 720 | 397 |
| 653 | 320 | 487 | 533 | 560 | 547 |
| 810 | 640 | 725 | 250 | 880 | 565 |
| 101 | 720 | 411 | 821 | 240 | 531 |
| 634 | 560 | 597 | 994 | 480 | 737 |
| 813 | 880 | 847 | 493 | 960 | 727 |
| 890 | 240 | 565 | 730 | 80 | 405 |
| 141 | 480 | 311 | 61 | 160 | 111 |
| 154 | 960 | 557 | 114 | 320 | 217 |
| 573 | 80 | 327 | 53 | 640 | 347 |
| 770 | 160 | 465 | 10 | 720 | 365 |
| 581 | 320 | 451 | 701 | 560 | 631 |
| 874 | 640 | 757 | 434 | 880 | 657 |
| 933 | 720 | 827 | 213 | 240 | 227 |
| 450 | 560 | 505 | 90 | 480 | 285 |
| 421 | 880 | 651 | 741 | 960 | 851 |
| 794 | 240 | 517 | 954 | 80 | 517 |
| 893 | 480 | 687 | 973 | 160 | 567 |
| 930 | 960 | 945 | 970 | 320 | 645 |
| 661 | 80 | 371 | 181 | 640 | 411 |
| 914 | 160 | 537 | 674 | 720 | 697 |
| 453 | 320 | 387 | 333 | 560 | 447 |
| 210 | 640 | 425 | 650 | 880 | 765 |
| 301 | 720 | 511 | 21 | 240 | 131 |
| 234 | 560 | 397 | 594 | 480 | 537 |
| 613 | 880 | 747 | 293 | 960 | 627 |
| 290 | 240 | 265 | 130 | 80 | 105 |
| 341 | 480 | 411 | 261 | 160 | 211 |
| 754 | 960 | 857 | 714 | 320 | 517 |
| 373 | 80 | 227 | 853 | 640 | 747 |
| 170 | 160 | 165 | 410 | 720 | 565 |
| 781 | 320 | 551 | 901 | 560 | 731 |
| 474 | 640 | 557 | 34 | 880 | 457 |
| 733 | 720 | 727 | 13 | 240 | 127 |
| 850 | 560 | 705 | 490 | 480 | 485 |
| 621 | 880 | 751 | 941 | 960 | 951 |
| 394 | 240 | 317 | 554 | 80 | 317 |
| 693 | 480 | 587 | 773 | 160 | 467 |
| 330 | 960 | 645 | 370 | 320 | 345 |
| 861 | 80 | 471 | 381 | 640 | 511 |
| 514 | 160 | 337 | 274 | 720 | 497 |
| 253 | 320 | 287 | 133 | 560 | 347 |
| 610 | 640 | 625 | 50 | 880 | 465 |
| 501 | 720 | 611 | 221 | 240 | 231 |

| | | | | | |
|---|---|---|---|---|---|
| 834 | 560 | 697 | 194 | 480 | 337 |
| 413 | 880 | 647 | 93 | 960 | 527 |
| 690 | 240 | 465 | 530 | 80 | 305 |
| 541 | 480 | 511 | 461 | 160 | 311 |
| 354 | 960 | 657 | 314 | 320 | 317 |
| 173 | 80 | 127 | 653 | 640 | 647 |
| 570 | 160 | 365 | 810 | 720 | 765 |
| 981 | 320 | 651 | 101 | 560 | 331 |
| 74 | 640 | 357 | 634 | 880 | 757 |
| 533 | 720 | 627 | 813 | 240 | 527 |
| 250 | 560 | 405 | 890 | 480 | 685 |
| 821 | 880 | 851 | 141 | 960 | 551 |
| 994 | 240 | 617 | 154 | 80 | 117 |
| 493 | 480 | 487 | 573 | 160 | 367 |
| 730 | 960 | 845 | 770 | 320 | 545 |
| 61 | 80 | 71 | 581 | 640 | 611 |
| 114 | 160 | 137 | 874 | 720 | 797 |
| 53 | 320 | 187 | 933 | 560 | 747 |
| 10 | 640 | 325 | 450 | 880 | 665 |
| 701 | 720 | 711 | 421 | 240 | 331 |
| 434 | 560 | 497 | 794 | 480 | 637 |
| 213 | 880 | 547 | 893 | 960 | 927 |
| 90 | 240 | 165 | 930 | 80 | 505 |
| 741 | 480 | 611 | 661 | 160 | 411 |
| 954 | 960 | 957 | 914 | 320 | 617 |
| 973 | 80 | 527 | 453 | 640 | 547 |
| 970 | 160 | 565 | 210 | 720 | 465 |
| 181 | 320 | 251 | 301 | 560 | 431 |
| 674 | 640 | 657 | 234 | 880 | 557 |
| 333 | 720 | 527 | 613 | 240 | 427 |
| 650 | 560 | 605 | 290 | 480 | 385 |
| 21 | 880 | 451 | 341 | 960 | 651 |
| 594 | 240 | 417 | 754 | 80 | 417 |
| 293 | 480 | 387 | 373 | 160 | 267 |
| 130 | 960 | 545 | 170 | 320 | 245 |
| 261 | 80 | 171 | 781 | 640 | 711 |
| 714 | 160 | 437 | 474 | 720 | 597 |
| 853 | 320 | 587 | 733 | 560 | 647 |
| 410 | 640 | 525 | 850 | 880 | 865 |
| 901 | 720 | 811 | 621 | 240 | 431 |
| 34 | 560 | 297 | 394 | 480 | 437 |
| 13 | 880 | 447 | 693 | 960 | 827 |
| 490 | 240 | 365 | 330 | 80 | 205 |
| 941 | 480 | 711 | 861 | 160 | 511 |
| 554 | 960 | 757 | 514 | 320 | 417 |
| 773 | 80 | 427 | 253 | 640 | 447 |
| 370 | 160 | 265 | 610 | 720 | 665 |
| 381 | 320 | 351 | 501 | 560 | 531 |

| Seed value=850 | | | Seed value=660 | | |
|---|---|---|---|---|---|
| **Modular** | **Chaos** | **Mixture** | **Modular** | **Chaos** | **Mixture** |
| 621 | 260 | 441 | 651 | 680 | |
| 394 | 520 | 457 | 284 | 640 | 666 |
| 693 | 960 | 827 | 763 | 720 | 462 |
| 330 | 80 | 205 | 740 | 560 | 742 |
| 861 | 160 | 511 | 691 | 880 | 650 |
| 514 | 320 | 417 | 804 | 240 | 786 |
| 253 | 640 | 447 | 523 | 480 | 522 |
| 610 | 720 | 665 | 620 | 960 | 502 |
| 501 | 560 | 531 | 131 | 80 | 790 |
| 834 | 880 | 857 | 524 | 160 | 106 |
| 413 | 240 | 327 | 883 | 320 | 342 |
| 690 | 480 | 585 | 300 | 640 | 602 |
| 541 | 960 | 751 | 971 | 720 | 470 |
| 354 | 80 | 217 | 444 | 560 | 846 |
| 173 | 160 | 167 | 843 | 880 | 502 |
| 570 | 320 | 445 | 780 | 240 | 862 |
| 981 | 640 | 811 | 211 | 480 | 510 |
| 74 | 720 | 397 | 564 | 960 | 346 |
| 533 | 560 | 547 | 403 | 80 | 762 |
| 250 | 880 | 565 | 60 | 160 | 242 |
| 821 | 240 | 531 | 851 | 320 | 110 |
| 994 | 480 | 737 | 884 | 640 | 586 |
| 493 | 960 | 727 | 563 | 720 | 762 |
| 730 | 80 | 405 | 140 | 560 | 642 |
| 61 | 160 | 111 | 891 | 880 | 350 |
| 114 | 320 | 217 | 404 | 240 | 886 |
| 53 | 640 | 347 | 323 | 480 | 322 |
| 10 | 720 | 365 | 20 | 960 | 402 |
| 701 | 560 | 631 | 331 | 80 | 490 |
| 434 | 880 | 657 | 124 | 160 | 206 |
| 213 | 240 | 227 | 683 | 320 | 142 |
| 90 | 480 | 285 | 700 | 640 | 502 |
| 741 | 960 | 851 | 171 | 720 | 670 |
| 954 | 80 | 517 | 44 | 560 | 446 |
| 973 | 160 | 567 | 643 | 880 | 302 |
| 970 | 320 | 645 | 180 | 240 | 762 |
| 181 | 640 | 411 | 411 | 480 | 210 |
| 674 | 720 | 697 | 164 | 960 | 446 |
| 333 | 560 | 447 | 203 | 80 | 562 |
| 650 | 880 | 765 | 460 | 160 | 142 |
| 21 | 240 | 131 | 51 | 320 | 310 |
| 594 | 480 | 537 | 484 | 640 | 186 |
| 293 | 960 | 627 | 363 | 720 | 562 |
| 130 | 80 | 105 | 540 | 560 | 542 |
| 261 | 160 | 211 | 91 | 880 | 550 |
| 714 | 320 | 517 | 4 | 240 | 486 |
| 853 | 640 | 747 | 123 | 480 | 122 |

| | | | | | |
|---|---|---|---|---|---|
| 410 | 720 | 565 | 420 | 960 | 302 |
| 901 | 560 | 731 | 531 | 80 | 690 |
| 34 | 880 | 457 | 724 | 160 | 306 |
| 13 | 240 | 127 | 483 | 320 | 442 |
| 490 | 480 | 485 | 100 | 640 | 402 |
| 941 | 960 | 951 | 371 | 720 | 370 |
| 554 | 80 | 317 | 644 | 560 | 546 |
| 773 | 160 | 467 | 443 | 880 | 602 |
| 370 | 320 | 345 | 580 | 240 | 662 |
| 381 | 640 | 511 | 611 | 480 | 410 |
| 274 | 720 | 497 | 764 | 960 | 546 |
| 133 | 560 | 347 | 3 | 80 | 862 |
| 50 | 880 | 465 | 860 | 160 | 42 |
| 221 | 240 | 231 | 251 | 320 | 510 |
| 194 | 480 | 337 | 84 | 640 | 286 |
| 93 | 960 | 527 | 163 | 720 | 362 |
| 530 | 80 | 305 | 940 | 560 | 442 |
| 461 | 160 | 311 | 291 | 880 | 750 |
| 314 | 320 | 317 | 604 | 240 | 586 |
| 653 | 640 | 647 | 923 | 480 | 422 |
| 810 | 720 | 765 | 820 | 960 | 702 |
| 101 | 560 | 331 | 731 | 80 | 890 |
| 634 | 880 | 757 | 324 | 160 | 406 |
| 813 | 240 | 527 | 283 | 320 | 242 |
| 890 | 480 | 685 | 500 | 640 | 302 |
| 141 | 960 | 551 | 571 | 720 | 570 |
| 154 | 80 | 117 | 244 | 560 | 646 |
| 573 | 160 | 367 | 243 | 880 | 402 |
| 770 | 320 | 545 | 980 | 240 | 562 |
| 581 | 640 | 611 | 811 | 480 | 610 |
| 874 | 720 | 797 | 364 | 960 | 646 |
| 933 | 560 | 747 | 803 | 80 | 662 |
| 450 | 880 | 665 | 260 | 160 | 442 |
| 421 | 240 | 331 | 451 | 320 | 210 |
| 794 | 480 | 637 | 684 | 640 | 386 |
| 893 | 960 | 927 | 963 | 720 | 662 |
| 930 | 80 | 505 | 340 | 560 | 842 |
| 661 | 160 | 411 | 491 | 880 | 450 |
| 914 | 320 | 617 | 204 | 240 | 686 |
| 453 | 640 | 547 | 723 | 480 | 222 |
| 210 | 720 | 465 | 220 | 960 | 602 |
| 301 | 560 | 431 | 931 | 80 | 590 |
| 234 | 880 | 557 | 924 | 160 | 506 |
| 613 | 240 | 427 | 83 | 320 | 542 |
| 290 | 480 | 385 | 900 | 640 | 202 |
| 341 | 960 | 651 | 771 | 720 | 770 |
| 754 | 80 | 417 | 844 | 560 | 746 |
| 373 | 160 | 267 | 43 | 880 | 702 |
| 170 | 320 | 245 | 380 | 240 | 462 |

| | | | | | |
|---|---|---|---|---|---|
| 781 | 640 | 711 | 11 | 480 | 310 |
| 474 | 720 | 597 | 964 | 960 | 246 |
| 733 | 560 | 647 | 603 | 80 | 962 |
| 850 | 880 | 865 | 660 | 160 | 342 |
| 621 | 240 | 431 | 651 | 320 | 410 |
| 394 | 480 | 437 | 284 | 640 | 486 |
| 693 | 960 | 827 | 763 | 720 | 462 |
| 330 | 80 | 205 | 740 | 560 | 742 |
| 861 | 160 | 511 | 691 | 880 | 650 |