

# Simulation Analysis Of Vehicle Four-Wheel Steering Control

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## ABSTRACT

In order to study the handling stability of four-wheel steering vehicle, the vehicle is simplified into a linear two-degree-of-freedom model. The fuzzy control technology is used to study the proportional feedforward + fuzzy feedback control strategy of four-wheel steering vehicle, and the fuzzy control toolbox in Matlab/Simulink is used to establish the four-wheel steering control system. The simulation results show that the fuzzy control can effectively improve the response of rear wheel steering, improve the maneuverability of the vehicle at low speed, reduce the turning radius, and improve the handling stability of the vehicle at high speed.

**Keywords:** four-wheel steering; handling stability; proportional feedforward; fuzzy control

## INTRODUCTION

With the improvement of people's living standards and the progress of science and technology, people's demand for cars is increasing year by year at the same time, the requirements for vehicles are increasingly high. On the one hand, the global environmental problems have made the automobile industry pay increasing attention to electric vehicles in recent years. On the other hand, drivers and passengers also put forward higher requirements for vehicle handling and safety. The birth of Four-Wheel Steering technology (4WS) greatly improves the steering stability and safety of the vehicle. At low speed, the front and rear wheels of the vehicle can realize reverse steering under certain control strategies, thus reducing the turning radius and greatly improving the steering flexibility of the vehicle. When the car needs to overtake at high speed, the car receives the information sent by the driver through the steering wheel and realizes the front and rear wheels rotate in the same direction under the control of the controller, thus making the car overtake smoothly and improving the safety of turning the steering wheel at high speed [1-2].

After three stages of development of four-wheel steering technology, various automobile research and development companies and universities have put forward many four-wheel steering control strategies. Compared with the traditional control method of front wheel steering, four wheel steering control can be roughly divided into three categories. The first one is the classic control mode and feedforward control of rear wheel steering, in which the driver's intention is estimated by the controller and the rear wheel steering is realized by the data table. Literature [3] proposed that the front and rear wheels of four-wheel steering should follow the steering proportion relationship, and this classic control can ensure that the side-deflection Angle of the center of mass is constant zero during steering. At the same time, a set of 4WS steering system was designed in literature [4] based on the rear-wheel Angle as a function between the speed and the front wheel Angle, which was controlled by single-chip microcomputer. Many Chinese scholars have also made a lot of research achievements in front and rear wheel proportional control.

For example, literature [5] conducted comparative analysis and theoretical research on two feedforward control methods for rear-wheel active steering vehicles. Based on the two-degree of freedom model of vehicles, steady-state response of rear-wheel active steering vehicles under front wheel angular step input conditions and continuous sinusoidal input conditions was analyzed. There is also a literature [6] that studies the integrated yaw torque control of vehicles with distributed four-wheel independent drive and independent steering. The front wheel steering Angle is obtained by responding to the driver's intention. The rear wheel steering Angle is controlled by feedforward proportional to the front wheel steering Angle. The proportional coefficient changes with the speed and other factors. In addition, many scholars use the form of feedforward + feedback to control four-wheel steering vehicles, aiming to develop a 4WS system with yaw speed gain similar to that of front-wheel steering vehicles. Literature [7] proposes the "Handling Modification System", Handling Modification System is a control scheme composed of feedforward, non-linear feedback and linear feedback items. In literature [8], considering the nonlinear characteristics of tires, the nonlinear three-step control strategy of active four-wheel linear steering vehicles is studied to improve the handling stability of four-wheel steering vehicles.

In this paper, a nonlinear three-step steering controller is proposed, which first obtains the theoretical sideslip Angle and yaw velocity based on the ideal vehicle model, and then compensates the influence of reference dynamics on the system by using feedforward control of reference signal changes. Tracking errors are minimized by state-dependent error feedback control to reduce system uncertainty and improve reliability. In recent years, the steering performance of four-wheel steering vehicles has been further improved by combining PID control, optimal control (LQ), LQG/LTR, neural network, fuzzy control, nonlinear control based on linear feedback, adaptive control, sliding film control and other control design methods [9].

In this era of continuous innovation of automobile technology, automobile industry changes with each passing day, intelligent driving will come, in order to achieve better four-wheel steering performance, to explore more effective control methods, four-wheel steering control technology for more in-depth understanding, careful discussion is very meaningful research. Based on this, this paper establishes a two-degree-of-freedom four-wheel steering vehicle model after comprehensively referring to various control strategies and combining with the parameter characteristics of the test vehicle itself, and establishes a four-wheel steering system based on proportional feedforward + fuzzy feedback control strategy in Matlab, and carries out simulation analysis. The results show that the feedforward + fuzzy control can effectively improve the handling stability of the vehicle.

**Establishment of ideal two degree of freedom vehicle model**

As a classical vehicle dynamics model, two degree of freedom vehicle stability model has been applied in many fields of vehicle dynamics. This paper uses this model to analyze the dynamic characteristics of four-wheel steering system and establish a basic four-wheel steering vehicle model, which is conducive to the establishment of high-performance four-wheel steering system control strategy. Figure 1 below is a schematic diagram of two-degree-of-freedom vehicle stability model:

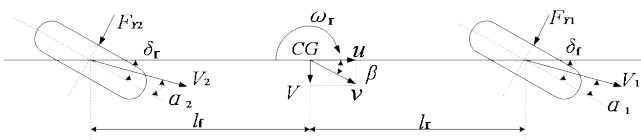


FIGURE 1: Two-degree-of-freedom operating model.

In the figure,  $F_{Y1}$  and  $F_{Y2}$  represent the reverse force generated by the front and rear wheels relative to the ground.  $\delta_f$  and  $\delta_r$  are rotation angles of the front and rear wheels of the vehicle;  $v$  is the component of the velocity of the center of mass on the Y-axis;  $u$  is the component of the velocity of the center of mass along the X-axis;  $l_f$  is the distance between the vehicle's center of mass and the front axle;  $l_r$  is the distance between the vehicle's center of mass and the rear axle;  $\omega_r$  is the yaw angular speed of the car at the center of mass;  $\alpha_1, \alpha_2$  are side Angle;  $\beta$  is the lateral declination Angle of vehicle at the position of center of mass.

According to Newtonian mechanical vector system, the dynamics equation of 4WS model with two degrees of freedom is obtained:

$$\sum F_y = F_{Y1} \cos \delta_f + F_{Y2} \cos \delta_r \quad (1)$$

$$\sum M_z = l_f F_{Y1} \cos \delta_f - l_r F_{Y2} \cos \delta_r \quad (2)$$

Since the wheel steering Angle is very small, it is approximately regarded as  $\cos \delta_f \approx 1$  and  $\cos \delta_r \approx 1$ . After the approximate substitution, equations (1) and (2) can be expressed as:

$$\sum F_y = k_1 \alpha_1 + k_2 \alpha_2 \quad (3)$$

$$\sum M_z = l_f k_1 \alpha_1 - l_r k_2 \alpha_2 \quad (4)$$

The side deflection Angle of the front and rear wheels of the car is:

$$\alpha_1 = \frac{v + l_f \omega_r}{u} - \delta_f \quad (5)$$

$$\alpha_2 = \frac{v - l_r \omega_r}{u} - \delta_r$$

Through the above formula, the relationship between the external force and external torque of the vehicle and the vehicle parameters can be obtained as follows:

$$\sum F_y = k_1 \left( \frac{v + l_f \omega_r}{u} - \delta_f \right) + k_2 \left( \frac{v - l_r \omega_r}{u} - \delta_r \right) \quad (6)$$

$$\sum M_z = l_f k_1 \left( \frac{v + l_f \omega_r}{u} - \delta_f \right) - l_r k_2 \left( \frac{v - l_r \omega_r}{u} - \delta_r \right)$$

Then, the differential equation of vehicle motion with two degrees of freedom can be written as:

$$\begin{cases} k_1 \left( \frac{v + l_f \omega_r}{u} - \delta_f \right) + k_2 \left( \frac{v - l_r \omega_r}{u} - \delta_r \right) = m(\dot{v} + u \omega_r) \\ l_f k_1 \left( \frac{v + l_f \omega_r}{u} - \delta_f \right) - l_r k_2 \left( \frac{v - l_r \omega_r}{u} - \delta_r \right) = I_z \dot{\omega}_r \end{cases} \quad (7)$$

In order to improve the modeling efficiency in Matlab/Simulink and make full use of the equation of state module, the vehicle differential equation of two degrees of freedom in the above equation is converted to the standard space equation as follows:

$$X' = AX + BU \quad (8)$$

$$Y = CX + DU$$

$$X = \begin{bmatrix} \beta \\ \omega_r \end{bmatrix}; Y = \begin{bmatrix} \beta \\ \omega_r \end{bmatrix}; U = \begin{bmatrix} \delta_f \\ \delta_r \end{bmatrix}; \quad (9)$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}; B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}; C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix};$$

$$a_{11} = \frac{k_1 + k_2}{mu}; a_{12} = \left( \frac{ak_1 - bk_2}{mu^2} - 1 \right); a_{21} = \frac{ak_1 - bk_2}{I_z}; a_{22} = \frac{a^2 k_1 + b^2 k_2}{I_z u};$$

$$b_{11} = -\frac{k_1}{mu}; b_{12} = -\frac{k_2}{mu}; b_{21} = -\frac{ak_1}{I_z}; b_{22} = \frac{bk_1}{I_z}$$

**RESEARCH ON FOUR WHEEL STEERING CONTROL STRATEGY**

**The establishment of proportional feedforward control module**

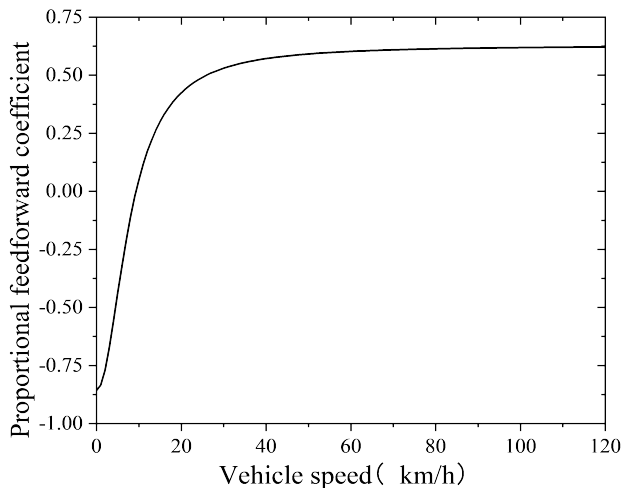
For the 4WS steering system, the system inputs are the front wheel Angle and the rear wheel Angle. Front wheel Angle is controlled by the driver (except steering-by-wire). Therefore, general 4WS steering adjusts the rear wheel Angle according to the front wheel Angle input to improve the vehicle's handling performance. Among the feedforward control strategies used by 4WS, the feedforward control strategy that takes steady-state lower side deflection Angle control as zero is relatively classical. This control strategy is a steering control idea in which the rear steering wheel calculates the rotation degree of the rear wheel according to the rotation Angle of the front steering wheel in the case of steering wheel input, and finally makes the steady-state side deflection Angle zero. According to the above analysis, the Angle relation between rear wheel Angle and front wheel Angle can be written as:

$$\delta_r = K_p \delta_f \quad (10)$$

Four-wheel steering control objectives in order to ensure the centroid side-slip Angle near the zero value, so the center of mass side-slip Angle is zero,  $\omega_r$  is constant, coupled with in this paper, we established two degree of freedom model before finishing available  $K_p$  is as follows:

$$K_p = -\frac{k_1(l_r^2 k_2 + l_r l_r k_2 + l_r m u^2)}{k_2(l_r^2 k_1 + l_r l_r k_1 - l_r m u^2)} \quad (11)$$

The basic parameters of the whole vehicle are substituted into the above formula to draw the curve of coefficient  $K_p$  with respect to  $u$  as shown in Figure 2.



**FIGURE 2:** Relation between proportional feedforward coefficient and vehicle speed.

It can be seen from Figure 2 that when the vehicle speed is low, the proportional coefficient of Angle of front and rear wheels is negative, that is, the wheels of front and rear steering wheels rotate in opposite directions. By reversing the front and rear wheels, the turning radius of the vehicle is smaller, the vehicle can complete the steering work in a smaller space, the steering is more flexible, and the steering maneuverability is further strengthened. When the driving speed of the vehicle exceeds the critical speed and needs to turn, the rotation Angle of the rear wheel and the rotation Angle of the front wheel become the same direction. Through such changes, the side-deflection Angle of the center of mass can be reduced and the steering and tracking ability of the vehicle can be improved, which is conducive to the stability and safety of the vehicle in high-speed lane change and overtaking conditions.

**Design of fuzzy feedback control for four wheel steering**

The design of fuzzy controller should take into account the complexity of fuzzy controller control process and control quality, so as to get a good application in engineering applications. Usually, two-dimensional fuzzy controllers are designed so as not to be particularly complicated in the design of fuzzy reasoning rules, so that the calculation time of fuzzy control is short and the accuracy of fuzzy control can be well adapted [11]. Therefore, when designing the fuzzy control of four-wheel steering, the difference between the output of the actual yaw velocity of the vehicle model and the output of the ideal yaw velocity of the reference model is used to obtain an input E of the fuzzy control. The difference change rate EC of the two is taken as another input, and the output of the fuzzy control is the compensation U of the rear wheel Angle. Finally, a two dimensional fuzzy controller with two inputs and one output is designed.

The design of fuzzy controller firstly carries on the fuzzy processing, namely carries on the discussion domain partition and fuzzy partition. The division of discourse domain is the artificial division of the continuous basic discourse domain into several discrete fuzzy discourse domain, also known as the quantification of discourse domain. The specific process is to divide the continuum domain [-a,a] into the discrete domain as {-n,-n+1... , 1, 1... ,n minus 1,n}. If the quantization series is too small in the selection of discrete domain, the expression of membership function in the discrete domain will not be accurate enough and the control effect will be poor. When the quantization series is too large, there will be too many single points used in the membership function in the calculation process, which will make the fuzzy control calculation process complicated. The sensitivity of fuzzy controller decreases with the decrease of quantization series, and the control effect is not detailed enough. And when we increase the quantization series infinitely, the domain tends to the continuum domain. Therefore, the fuzzy subsets of input and output are divided into seven language variables in this paper. The domain value of input variable E of yaw velocity error is set as [-6,6]; The theoretical domain value of the error change rate EC of the yaw velocity of the input variable is [-10,10]. The fuzzy domain of output variable to compensate rear wheel Angle is defined as [-1,1]. The quantization factor  $K_e, K_{ec}$  and scale factor  $K_u$  used in the quantization process are calculated as follows:

$$K_e = n / x_e \quad , \quad K_{ec} = m / x_{ec} \quad , \quad K_u = l / x_u \quad (12)$$

The fuzzy controller to be designed is a dual-input single-output controller, and the control rules are expressed by the following fuzzy conditional statements :If E is NB and EC is NB then U is PB. According to the control experience, fuzzy rules are preliminarily formulated, and then appropriately adjusted according to the control effect after simulation. The final fuzzy rules are determined as follows.

**TABLE 1:** Fuzzy control rule table.

E \ EC	EC						
	NB	NM	NS	Z0	PS	PM	PB
NB	PB	PB	PB	PM	PS	Z0	NS
NM	PB	PB	PB	PM	PS	Z0	NS
NS	PB	PB	PM	PS	Z0	NS	NM
Z0	PB	PM	PS	Z0	NS	NM	NB
PS	PM	PS	Z0	NS	NM	NB	NB
PM	PS	Z0	NS	NM	NB	NB	NB
PB	Z0	NS	NM	NB	NB	NB	NB

There are 11 membership functions in matlab/Simulink for users to choose independently according to different needs. In the fuzzy control designed in this paper, membership function designs of three variables, namely yaw velocity difference E, yaw velocity difference change rate EC and rear wheel Angle compensation change U, are shown in Figure 3. - Figure 5., Figure 6 shows the MAP of fuzzy control rules.

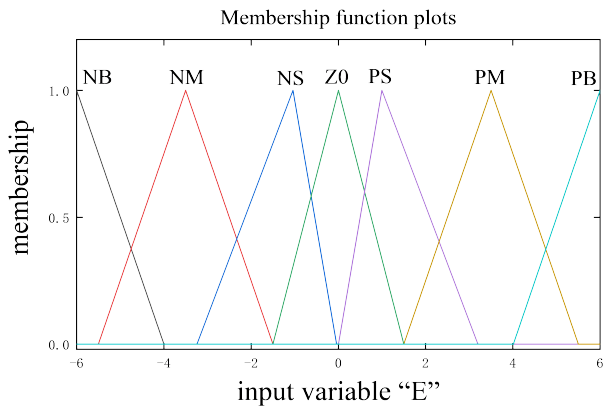


FIGURE 3: Fuzzy control input variable yaw velocity difference E.

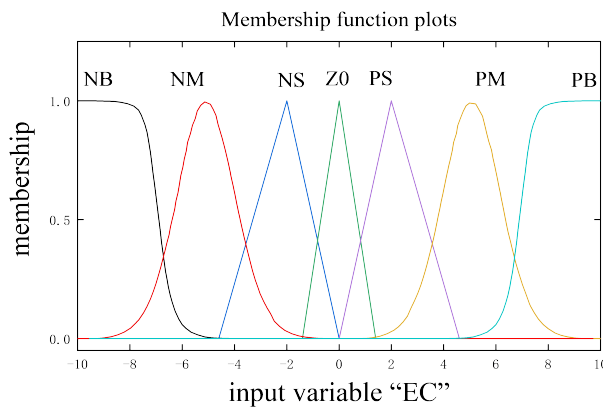


FIGURE 4: Fuzzy control input variable yaw velocity difference rate EC.

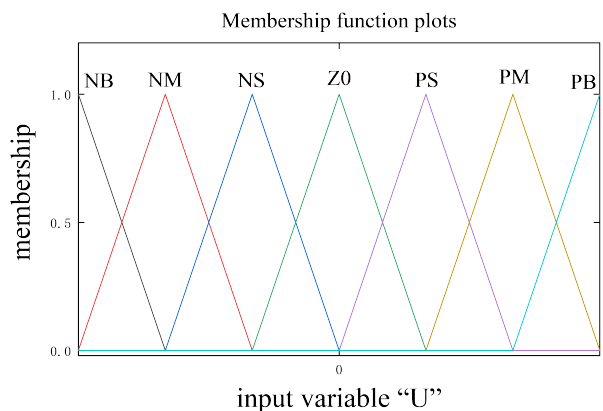


FIGURE 5: Fuzzy control output variable rear wheel Angle compensation value U.

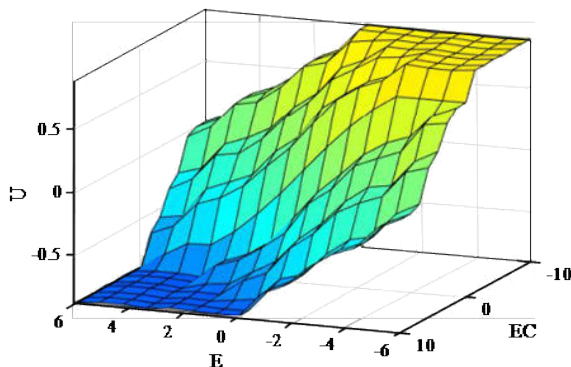


FIGURE 6: MAP of fuzzy control rules.

**Design of fuzzy feedback reference model**

The expression of the two-degree-of-freedom four-wheel steering model has been derived in the previous paper. For equation (7), Laplace transform should be carried out using Matlab symbol toolbox and put into the initial condition 0 to obtain the yaw velocity transfer function formula with the front wheel rotation Angle:

$$\frac{\omega_z}{\delta_f}(s) = \frac{ml_l k_1 \cdot s + Lk_1 k_2}{ml_z \cdot s^2 - [L_z(k_1 + k_2) + m(l_z^2 k_1 + l_z^2 k_2)] \cdot s + \frac{k_1 k_2}{u} L^2 \left[ 1 + \frac{mu^2(l_z k_2 - l_z k_1)}{k_1 k_2 L^2} \right]} \tag{13}$$

Under the condition that the side deflection Angle of the center of mass tends to be 0, yaw velocity is the first-order hysterical response of the rotation Angle of the front wheel. The yaw velocity response in the above equation is simplified into the following standard form:

$$\frac{\omega_z}{\delta_f}(s) = G_{\omega_z} \frac{1}{1 + T_e \cdot s} \tag{14}$$

Where  $G_{\omega_z} = \frac{u}{(1 + Ku^2)L}$ ,  $K$  is the stability factor:

$$K = \frac{m}{L^2} \left( \frac{l_z}{k_2} - \frac{l_z}{k_1} \right), \text{ and } T_e \text{ is the first order inertial time}$$

$$\text{constant: } T_e = \frac{1}{\sqrt{\frac{L^2 k_1 k_2}{ml_z u^2} (1 + Ku^2)}}$$

Since the lateral deflection Angle of the center of mass is 0 under the set ideal state, the transfer function of the lateral deflection Angle of the center of mass to the Angle of the front wheel can be expressed as:

$$\frac{\beta}{\delta_f}(s) = G_{\beta} = 0 \tag{15}$$

The ideal transfer function of the reference model can be expressed as:

$$X_1(s) = \begin{bmatrix} \beta_1 \\ \omega_{r1} \end{bmatrix} = \begin{bmatrix} G_{\beta_1} \\ G_{\omega_{r1}} \frac{1}{1 + T_e \cdot s} \end{bmatrix} \delta_f(s) \tag{16}$$

Rewrite the above equation into an equation of state as follows:

$$X_1' = A_1 X_1 + \delta_f B_1 \tag{17}$$

$$X_1 = \begin{bmatrix} \beta_1 \\ \omega_{r1} \end{bmatrix}; A_1 = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{T_e} \end{bmatrix}; B_1 = \begin{bmatrix} 0 \\ \frac{G_{\omega_{r1}}}{T_e} \end{bmatrix}$$

**Comparative simulation analysis of vehicle four-wheel steering**

In the process of turning, the driving speed of the vehicle itself is the main factor that determines whether the vehicle can turn smoothly. The centrifugal force of vehicle steering is affected by the speed and turning radius, and whether it can roll over is also directly related to the speed. In order to study the steering process of the vehicle under the control of the designed steering controller, the simulation was set according to the actual situation. In this simulation, low, medium and high speed were adopted, with the speed being 40km/h, 80km/h and 120km/h respectively.

The angular step simulation under different speeds was conducted for the vehicle controlled by two-wheel steering and four-wheel steering under no control state, and the simulation results were recorded to obtain the corresponding values of the running parameters of the vehicle, such as centroid side deflection Angle, yaw velocity and lateral acceleration, as shown in Figure 7 - Figure 9 below.

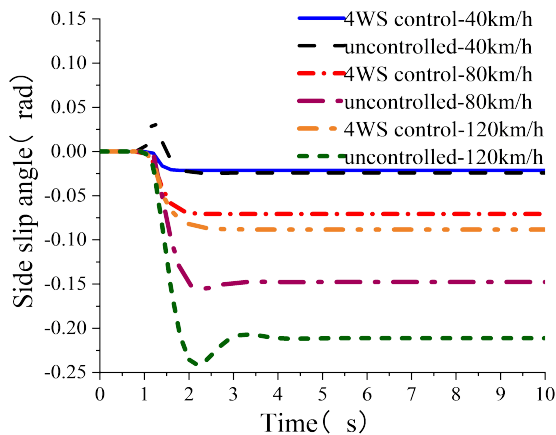


FIGURE 7: Side slip angle at different vehicle speeds.

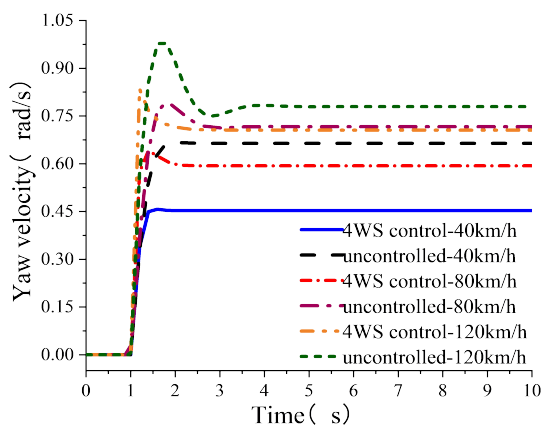


FIGURE 8: Yaw velocity at different vehicle speeds.

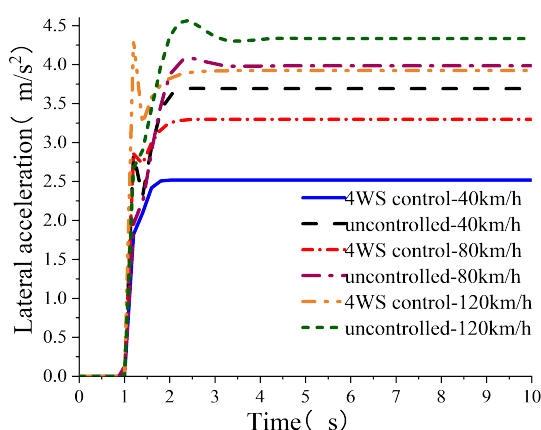


FIGURE 9: Lateral acceleration at different vehicle speeds.

No matter at low speed or high speed, the parameters of lateral yaw Angle, yaw speed and lateral acceleration of the vehicle controlled by the designed four-wheel steering are better than those of the vehicle without control. It can be seen from Figure 7 that under the control of the designed four-wheel steering, the simulation results of centroid side deflection Angle tend to increase gradually with the increase of the speed, and the increase rate of change gradually decreases, which is conducive to inhibiting the

instability of the vehicle due to the change of the speed and wheel Angle. Among them, when the speed is low, the vehicle's centroid side Angle can be controlled near 0, and when the speed is high, the maximum centroid side Angle is -0.08rad, relative to the front wheel steering vehicle 4WS centroid side Angle value reduced by 58.16%, the general car's side Angle is within 5.3°, It can be considered that the relationship curve between side deflection Angle and side deflection force is linearly correlated, which ensures that the driver has a good control over the vision of the vehicle heading direction. As can be seen from Fig. 8, under the driving state of 120km/h, the maximum yaw speed of the four-wheel steering vehicle with the control strategy is 0.7rad/s, which is 10.46% lower than that under the control state. It can be seen from Fig. 9 that the lateral acceleration of the vehicle also gradually increases with the increase of the vehicle speed. It is generally believed that the vehicle stability is better when the maximum lateral acceleration is less than 0.4g. However, the 4WS vehicle has a small overshoot at 120km/h, but it quickly reaches 3.9m/s<sup>2</sup>. Therefore, the design of four-wheel steering control can fully meet the steering needs in general conditions.

SUMMARY

In this paper, the proportional feedforward + fuzzy feedback 4WS control research, simulation verification. 4WS technology is an important part of modern chassis control technology, which can effectively improve the driving performance of vehicles. The simulation results show that the maximum center-of-mass side Angle of 4WS vehicle using proportional feedforward + fuzzy feedback is -0.08rad compared with the front-wheel steering vehicle, and the center-of-mass side Angle of 4WS vehicle is reduced by 58.16% compared with the front-wheel steering vehicle. The maximum yaw velocity is 0.7rad/s, which is 10.46% lower than that in the uncontrolled state. Even at the maximum speed of 120km/h, the lateral acceleration is less than 0.4g and 3.9m/s<sup>2</sup>. The adoption of four-wheel steering can improve the stability and tracking of vehicles, further improving the driver's driving experience on better road conditions and the safety of driving on poor roads. The proportional feedforward + fuzzy feedback four-wheel steering control strategy designed in this paper can well improve the steering stability and flexibility of vehicles at different speeds, improve the vehicle's tracking and stability, and make the driver more relaxed and comfortable when driving the vehicle.

REFERENCE

- [1] Wu Lei, Wang Qi, Zhang Peng. Research on Vehicle Stability Based on Four Wheel Steering and Direct Yaw Moment [J]. Machinery Design & Manufacture,2023(02):32-36.
- [2] Zhang Jia-xu, ZHOU Shi-ying, LI Chang-long, Guo Chong, ZHAO Jian. Path Planning of Autonomous Valet Parking for Drive-by-wire Four-Wheel Steering Vehicles [J]. China Mechanical Engineering, 2022,33(08):993-1000.
- [3] Shoichi Sano, Yoshimi Furukawa, Shuji Shiraishi. Four Wheel Steering System with Rear Wheel Steer Angle Controlled as a Function of Steering Wheel Angle [J]. SAE Technical Paper Series, 1986.
- [4] Y Shibahata, N Irle, H Itoh. The Development of an Experimental Four-Wheel Steering Vehicle [J]. SAE 860623, 1986.
- [5] Cui Gaojian, Li Bin, Li Shaosong, et al. Comparative Analysis of Feedforward Controller for Rear-wheel Active Steering Vehicle [J]. Mechanical Engineering and Automation, 2018,207(2):31-33.

- [6] Lei Yongqiang. Research on Integrated Control of Four-Wheel Steering and Yaw Moment for Independent Steering/Independent Electric Drive Vehicle [D]. Jinzhou: Liaoning University of Technology, 2017
- [7] RUSSELL H E B, GER56DES J C. Design of Variable Vehicle Handling Characteristics Using Four-Wheel Steer-by-Wire [J]. IEEE Transactions on Control Systems Technology, 2016, 24 (5): 1529-1540.
- [8] Yu S, LiW, WangW, et al. Nonlinear Control of Active Four Wheel Steer-By-Wire Vehicles [J]. IEEE Access,2019(8):1-11.
- [9] Luo Yutao, Zhou Tian-yang, Xu Xiao-tong. Time-varying LQR Control for four-wheel Steering-Drive Vehicle based on Genetic Algorithm [J]. Journal of South China University of Technology (Natural Science Edition), 2021, 49(03):114-122.
- [10] Zhu Shaopeng, Wei Bangxuan, Liu Dong, Chen Huipeng, Huang Xiaoyan, Zheng Yingjie, Wei Wei. A Dynamics Coordinated Control System for 4WD-4WS Electric Vehicles [J]. Electronics, 2022, 11(22):3731.
- [11] Wang Siwen. Research on Four-Wheel Steering Modeling Simulation and Control Strategy Based on an Off-road Vehicle [D]. Shiyan: Hubei University of Automotive Industry, 2017.