

The Physical Nature of Time Dilation in The Hafele-Keating Experiment and In GPS Satellite Clocks

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ABSTRACT

The reason for the change in the time course in the clocks of the Hafele-Keating experiment and GPS satellites is not relativistic effects, but an increase in the path length due to the displacement of the perihelion of the satellite's orbit at the corresponding flight altitude. Based on Kepler's laws, are derived formulas for calculating corrections to the duration of the rotation period. The results of calculations based on these formulas coincide with the predictions of the theory of relativity and with the data of observations and experiments. The correspondence of the calculation results to the observations in the Hafele-Keating experiment when airplanes move in the east and west directions is achieved on the assumption that the displacement of the perihelion orbits does not occur in the direction of motion, as is customary in the theory of relativity, but in the direction of rotation of the center of gravity. This assumption, which was also confirmed when calculating corrections for the duration of the rotation period of GPS satellites, allows us to conclude that the cause of the perihelion shift is the rotation of the center of gravity. It is established that relativistic effects, including the displacement of the perihelion orbits, which are not described by the Newtonian theory of gravity, are a consequence of the influence of the angular momentum of the rotating center of gravity. If the center of gravity does not rotate, then there are no relativistic effects, and all movements of material bodies in the gravitational field occur in accordance with Newton's classical theory of gravity. Movements in the gravitational field of a rotating center of gravity are described by Newton's updated theory of gravity, which takes into account the influence of angular momentum, not just the mass of the center of gravity. The influence of the rotation of space objects in the theory of relativity is also taken into account (the Sagnac effect). But this effect is insignificant; the corrections to the clock in GPS satellites, for example, are three orders of magnitude smaller than the corrections from other effects. In Newton's updated theory of gravity, as in the theory of relativity, time is not absolute. The passage of time depends both on the speed of movement along the orbit and on the distance to the center of gravity, which is confirmed by the coincidence of the calculation results with the clock readings in the Hafele-Keating experiment and in GPS satellites.

Keywords: gravitational theory; twin paradox; perigee shift; GPS satellites; center of attraction

THE HAFELE-KEATING EXPERIMENT

The Hafele-Keating experiment [1] is one of the tests of the theory of relativity and is considered to be a demonstration of the reality of time dilation for moving objects, as well as the twin paradox. Despite the difficulties in conducting the experiment, it was performed at a high theoretical and technical level, which is confirmed by the results and their reliability assessments. Since the moving object was a plane traveling around the world, the question arises as to whether the results of the experiment are related to the very first test of the theory of relativity, which is the anomalous perihelion shift of Mercury, which is also a traveling object around the world.

In 1915, Einstein wrote: "The result concerning the perihelion shift of Mercury fills me with deep satisfaction" [2]. Since the perihelion shift of an orbit increases the length of the path and the period of revolution, there is every reason to assume that this can explain the change in flight time in the Earth's gravitational field.

The relativistic effects considered in the theoretical part of the Hafele-Keating experiment include the constancy of the speed of light, the equivalence principle, time dilation, gravitational frequency shifts, and others. Since the predicted perihelion shift of Mercury's orbit matches the observation with high accuracy, it can be assumed that the influence of these effects is also taken into account in the formula for calculating this shift over one revolution, $S = 6\pi GM/c^2$ [3]. The peculiarity of this formula is that the linear shift S is the same for all planets and does not depend on their mass or distance from the attracting center. This formula can also be used to calculate the perihelion shift of Earth's satellite orbits. For example, the observed and confirmed angular shift of the Moon's perihelion is 0.06" over 100 Earth years [3]. If we convert this to a linear shift, we find that the linear shift of the Moon's perihelion over one revolution is 0.0834 m. Since the Moon has a diameter of 3,500 kilometers, the perihelion shift is 0.00000024 of the diameter.

This figure can be used to estimate the size of the anomalous gravitational effects that are not described by Newton's classical theory of gravity, as well as the corrections that relativistic effects introduce into gravitational calculations. These corrections can be calculated in the Hafele-Keating experiment using the method described in [1], or the known value of S can be used to estimate the change in orbital period due to the shift in the perigee of the orbit during a round-the-world journey.

If we consider the motion of an artificial satellite (AS) around the Earth, then the perigee shift of the orbit will also be the same as that of the Moon - 0.0834 m. The perigee shift of the orbit does not mean that the orbit itself is elongated. It simply rotates by an angle corresponding to the shift. In the theory of relativity, it is assumed that the perigee shift is a consequence of the satellite's motion in the orbit, and its direction coincides with the direction of the satellite's velocity and does not depend on the direction of rotation of the attracting center. Observations of the perigee shifts of the planets in the Solar System indicate that they all shift in the direction of the Sun's rotation. In [3], it is shown that the moment of momentum of a rotating mass creates tangential forces in the surrounding space, which cause the perigee shifts of the satellites' orbits and are directed in the direction of the attracting center's rotation. If this is confirmed by calculations and observations, then the perigee shift of Mercury ceases to be an observational confirmation of the theory of relativity.

For calculations and comparison with predictions and results in the Hafele-Keating experiment, let's consider a non-stop journey to the east and west at the level of the 32-nd parallel, with the altitude, average speed, and duration provided in [1].

The simplest formula for calculating the correction to the duration of a round-the-world journey is the ratio of the perigee shift to the absolute velocity of the aircraft's motion, S/v_a , in the locally inertial coordinate system (ECI). However, this correction, like the perigee shift, does not depend on the altitude of the flight. The task of accounting for this dependence involves comparing the flights of the AS at the Earth's level and at an altitude of h.

The second cosmic velocity at which the AS flies depends on the altitude of the flight. The AS-1 flies at an altitude of h with a speed of:

$$v_1 = \sqrt{GM / (r + h)} ;$$

Here, G is the gravitational constant, M and r are the mass and radius of the Earth, and h is the distance to the Earth's surface. The AS-2 flies on the Earth's surface at the airport level at a speed of:

$$v_2 = \sqrt{GM / r} ;$$

The time required for the satellites to travel a distance S is:

$$t_1=S/v_1; t_2=S/v_2; (1)$$

The time $t_1 - t_2$ is the change in the duration orbital period AS-1 compared to the AS-2:

$$t_{12} = (t_1 - t_2) = S (1-\sqrt{r_e / (r_e + h)}) / v_1;$$

The absolute speed of the aircraft can be represented as the sum of the Earth's rotational speed v_e and the aircraft's relative speed v:

$$v_a = v_e + v$$

Since the aircraft's absolute speed v_a is significantly lower than the satellite's speed v_1 , the change in the aircraft's orbital period is proportionally greater and can be calculated using the formula:

$$t_a = t_{12} v_1 / (v_e + v); (2)$$

If we assume $v = 0$ in this formula, we obtain a universal formula for calculating the time change based on altitude for objects that are stationary relative to the Earth:

$$t_e = t_{12} v_1 / v_e = S (1-\sqrt{r / (r + h)}) / v_e; (3)$$

In the Hafele-Keating experiment, the time correction of a clock that has completed a round-the-world journey is compared to a clock that is stationary relative to the Earth. This means that the result of the absolute speed (2) must be subtracted from the result related to the Earth's rotation (3):

$$t(v) = t_a - t_e = -S (1-\sqrt{r / (r + h)}) v / (v_e + v) / v_e; (4)$$

Figure 1 shows a graph of the time correction for a round-the-world journey as a function of the aircraft's relative speed. In the upper-right corner, there is a similar graph from [1] that is constructed in a coordinate system that rotates with the Earth.

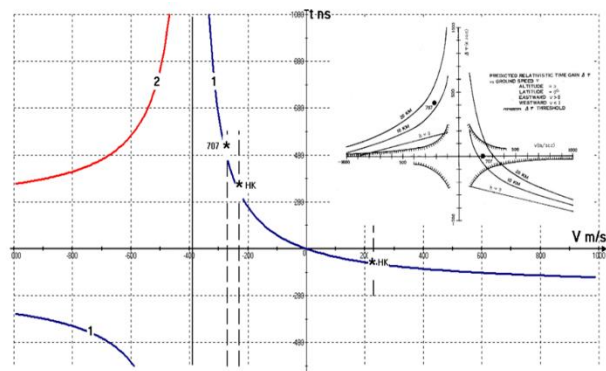


FIGURE 1: Dependence of the correction to the duration of a round-the-world trip on the speed of the aircraft in the Hafele-Keating experiment. In the upper right corner is a similar graph from [1] constructed in a coordinate system that rotates with the Earth.

The calculated corrections to the period of revolution of a non-stop round-the-world trip for movement in the western and eastern directions (265 ns; -45 ns) coincide with the result of the forecast for the Hafele-

Keating experiment (275 ns; -40 ns). The Boeing 707's change in the period of revolution is also a coincidental result in these figures.

Figure 1 also clarifies the question of whether the displacement of the perihelion depends on the Earth's rotation. If the aircraft is moving in the eastern direction, then the direction of the aircraft's movement, the direction of the perihelion's displacement, and the direction of the Earth's rotation coincide. If the aircraft is moving in the western direction and its speed approaches the Earth's rotation speed, then the period of revolution and the corrections to the period of revolution are positive and increase to infinity. From the graph of the $t(v)$ function, it follows that if the aircraft is moving in the western direction and its speed is greater than the Earth's rotation speed, then the corrections to the period of revolution (line 1) calculated using formula (4) are negative. This can only be corrected by changing the sign of S in formula (4). In this case, the continuation of the $t(v)$ graph in the western direction would be line 2. However, this change of sign means that the length of the path from perihelion to perihelion in the western direction decreases, which means that the perihelion of the aircraft's orbit shifts towards the aircraft, or towards the Earth's rotation. Therefore, the direction of the perihelion's displacement in both cases coincides with the direction of the Earth's rotation, rather than with the direction of the aircraft's movement.

DAILY CORRECTIONS TO THE ORBITAL PERIOD OF GPS SATELLITES

The calculation and control of time corrections is relevant for GPS satellites, which also make round-the-world trips in a geocentric orbit at a given altitude and in various directions relative to the Earth's equatorial plane. In the theory of relativity, these corrections are considered relativistic, and the formula that includes gravitational and kinematic corrections for satellite orbits is as follows [4]:

$$t(h) = T \frac{GM}{c^2} \frac{r-2h}{2r(r+h)}; (5)$$

Here, h is the distance to the Earth's surface, T is the orbital period of the satellite, and c is the speed of light.

This formula is used to calculate corrections to the orbital period of the satellite based on the distance to the Earth (Figure 2, red line 1). This formula can also be used to calculate corrections for a specific time τ by assuming $T = \tau$ (Figure 2, red line 2).

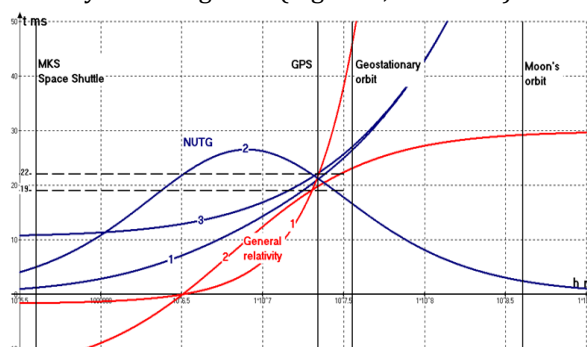


FIGURE 2: Dependence of time corrections on the Earth's distance for an artificial satellite according to the theory of relativity and the updated Newton's theory of gravity (NUTG). Lines 1 represent time corrections for the orbital period T , while lines 2 represent time corrections for 12 hours (43200 seconds).

In the updated Newton's theory of gravity (HUTG), the simplest formula for calculating time corrections for the orbital period of an artificial satellite for any distance from the Earth's center is similar to formula (1):

$$t(h) = S/v_1; (6)$$

In Figure 2, this function is represented by the blue line 3, which is non-zero up to the Earth's center. These are the absolute time corrections for satellites at any distance from the Earth's center.

In observations, the clock's time on orbit is compared with the clock's time on Earth, so the corrections are the difference between these values. To ensure that they are zero on the Earth's surface in HUTG, they are calculated using a formula similar to (4):

$$t(h) = S \left(\frac{1}{v_1} - \frac{1}{v_2} \frac{T_2}{T_1} \right); (7)$$

The coefficient T_2/T_1 provides the correction for the clock of SA-2 during the orbital period of SA-1. By substituting the known values of the parameters into formula (7), we obtain a formula for calculating time corrections for the orbital period of an artificial satellite based on the distance to the Earth:

$$t(h) = S \left(1 - \frac{r^2}{(r+h)^2} \right) \sqrt{\frac{r+h}{GM}}; (8)$$

In Figure 2, this function is represented by the blue line 1.

To calculate the corrections not for the period of revolution T , but for a specific time τ , it is necessary to multiply the correction of each satellite by the coefficient τ/T_i in formula (7).

$$t(h) = S \left(\frac{1}{v_1} \frac{\tau}{T_1} - \frac{1}{v_2} \frac{T_2}{T_1} \frac{\tau}{T_2} \right) = S \left(\frac{1}{v_1} - \frac{1}{v_2} \right) \frac{\tau}{T_1}; (9)$$

Substituting the known values of the parameters into formula (9), we obtain a formula for calculating the corrections to the satellite clock for a specific time τ , depending on the distance to the Earth:

$$t(h) = S \left(1 - \sqrt{\frac{r}{r+h}} \right) \frac{\tau}{2\pi(r+h)}; (10)$$

In Figure 2, this function is represented by the blue line 2.

As can be seen from Fig. 2, observations of GPS satellite clocks confirm both the corrections calculated using Einstein's theory of relativity and

the updated Newtonian theory of gravity. When converted to Earth's day (24 hours), the theoretical corrections range from 38 to 45 ms. For satellites with orbits at different distances from Earth, the results of calculations based on these theories differ significantly. This includes space stations, geostationary satellites, and clocks on the Moon. Given the vast number of satellites orbiting Earth, it is surprising that there are no other observations in the literature that support the theoretical predictions of the theory of relativity, other than those obtained from GPS satellite clocks.

Formulas (6), (7), and (9) for calculating corrections to the orbital period of GPS satellites provide further clarification on the direction of the perigee shift. To ensure that the results of calculations do not depend on the direction of motion, it is necessary that these results do not change when the signs of the velocities in the formulas are reversed. However, if the signs of the velocities v_i are replaced by $-v_i$ in these formulas, the corrections become negative. To ensure that the results of calculations based on these formulas do not change when the signs of the velocities are reversed, it is sufficient to replace S with $-S$. Changing the sign of the velocity represents a change in the direction of motion, while changing the sign of the perigee shift represents a change in the length of the path. If a satellite is moving eastward, the perigee shift is positive, and the length of the path from perigee to perigee increases. If a satellite is moving westward, the perigee shift is negative, and the length of the path from perigee to perigee decreases. Therefore, the perigee of satellite orbits shifts in the direction of Earth's rotation, rather than in the direction of the velocity. However, the calculated corrections to the orbital period remain consistent regardless of the direction of motion of the satellites.

Thus, the results of the Hafele-Keating experiment and observations of GPS satellites can be predicted using both the theory of relativity and the updated Newtonian theory of gravity. However, in the theory of relativity, the corrections to the duration of the orbital period are considered to be relativistic effects that confirm the slowing down of time and the twin paradox, while in Newton's updated theory of gravity, they are attributed to changes in the length of the path as a result of the shift in the perihelion of the orbit due to the momentum of the rotating Earth.

CONCLUSION

In the theory of relativity, the results of the Hafele-Keating experiment and the daily time corrections in the GPS satellites' clocks are considered to be relativistic effects that affect the clock's ticking due to their movements in the Earth's gravitational field. The peculiarity of these movements is that they occur in circular Keplerian orbits, for which it is known that their perihelion shifts, increasing the length of the path per revolution. Assuming that these changes also lead to a change in the duration of a round-the-world journey, the article derives formulas for calculating gravitational corrections to the orbital period. The results of calculations using these formulas coincide with the predictions of the theory of relativity and

with the data from observations and experiments.

The Hafele-Keating experiment, which involves a round-the-world journey in both directions, also revealed that the shift in the perihelia of planets and satellites occurs not in the direction of their orbital motion, as is commonly accepted in the theory of relativity, but in the direction of the rotation of the center of gravity. This non-trivial result, which is also confirmed by calculations for GPS satellites, serves as a test for validating the proposed approach to calculating gravitational time corrections.

Elementary formulas have been proposed for calculating the impact of the shift in the perihelia of artificial satellites' orbits on the gravitational time corrections in GPS, both relative to the center and relative to the Earth's surface. It has been established that the results of these calculations do not depend on the direction of the satellites' motion. Given that these results coincide with the predictions of the theory of relativity for the GPS satellites' orbits, additional experimental studies and observations are required to verify the methods. Based on the results of this work, it can be concluded that relativistic effects, including the shift in the perihelia of orbits that are not described by Newton's theory of gravity, are a consequence of the influence of the moment of momentum of the rotating center of gravity. All movements in the gravitational field of a rotating center of gravity are described by Newton's updated theory of gravity [5]. If the center of gravity does not rotate, then relativistic effects are absent, and all movements in the gravitational field occur in accordance with Newton's classical theory of gravity.

In Newton's updated theory of gravity, as in the theory of relativity, time is not absolute. The flow of time depends on both the speed of orbital motion and the distance to the center of gravity, which is confirmed by the coincidence of the calculation results with the readings of the clocks in the Hafele-Keating experiment and in GPS satellites.

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